Linguistic Mapping

The Principles of Calculus I

IV

Symmetry

IV.4

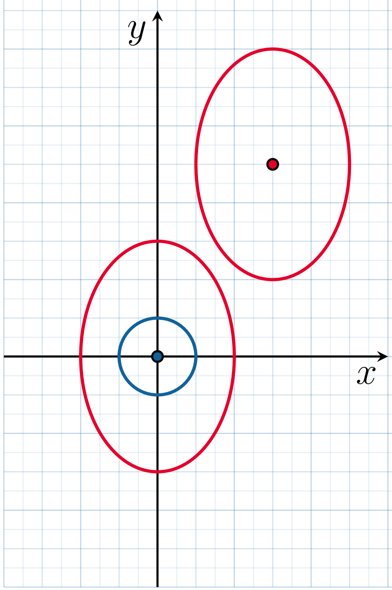
Symmetry of Tangential Intersections

Classroom Exercises

Take E to be the ellipse that is the solutions set to this equation:

$$\frac{(x-3)^2}{4} + \frac{(y-5)^2}{9} = 1.$$

(a) Identify a composite Φ of asymmetric scalings and a translation that transforms the unit circle $\mathscr{C}$ into E.

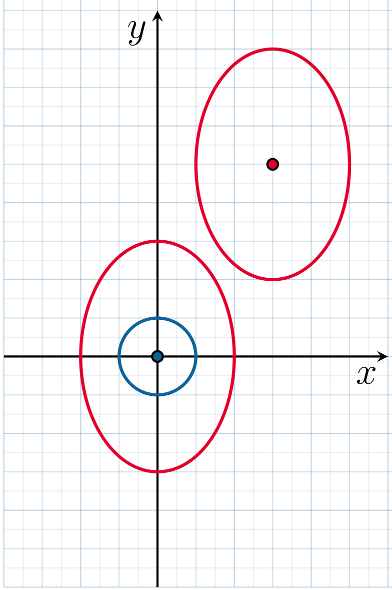


Take E to be the ellipse that is the solutions set to this equation:

$$\frac{(x-3)^2}{4} + \frac{(y-5)^2}{9} = 1.$$

(b) For any point (a, b) in E, identify a point p in $\mathscr{C}$ so that

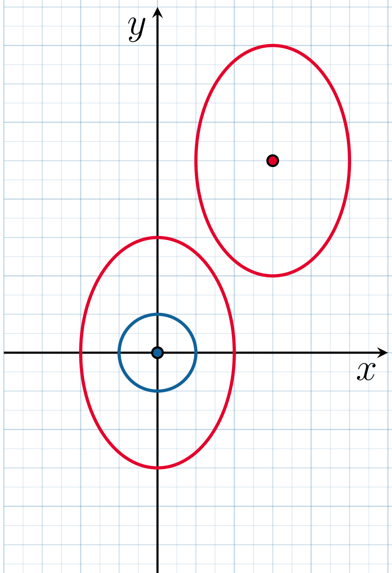
$$\Phi(p) = (a, b).$$



Take E to be the ellipse that is the solutions set to this equation:

$$\frac{(x-3)^2}{4} + \frac{(y-5)^2}{9} = 1.$$

(c) Determine an equation for the line tangent to E at (a, b).



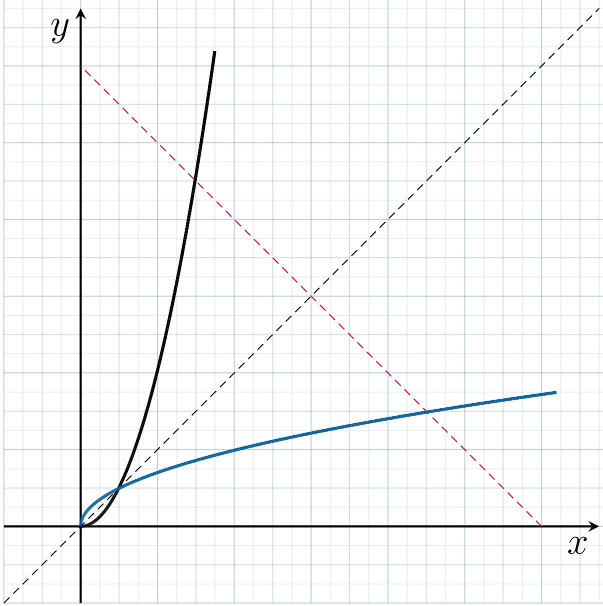
Exercise 2

Take f to be the restriction of pow_2 to $[0, \infty)$. The function f is invertible and pow_2^{-1} is the square root function.

(a) Determine the line L that is tangent to f at $(3, 9)$.

(b) Reflect L and f across pow_1 to obtain L^{-1}, f^{-1}, and their intersection at $(9, 3)$. Identify a formula for the line that is tangent to f^{-1} at $(9, 3)$.

(c) Determine $(f^{-1})'(9)$.



Take f to be the function given by

$$f(x) = \frac{x}{2x-3}.$$

Note that $(3,1)$ is in f.

(a) Determine an equation for the line L that is tangent to f at $(3,1)$.

(b) Evaluate $(f^{-1})'(1)$ by reflecting L across pow_1.

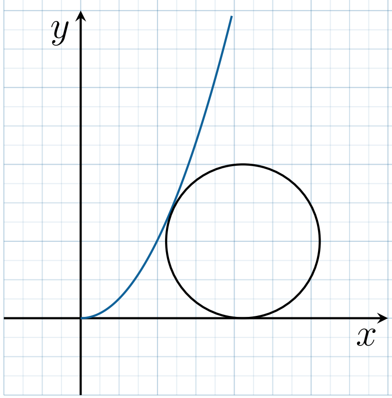
(c) Evaluate $(f^{-1})'(1)$ directly by identifying a formula for f^{-1}.

For any natural number n and any positive real number a, use the reflection across pow_1 to determine an equation for the line that is tangent to $\operatorname{pow}_{\frac{1}{n}}$ at (a^n, a). Determine an equation for $\operatorname{pow}'_{\frac{1}{n}}$ on $(0, \infty)$.

Exercise 5

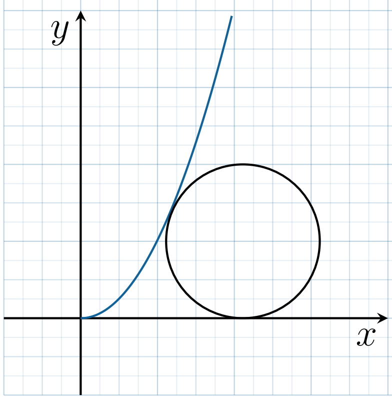
The lowest point of a circle lies on the x-axis, the circle is to the right of the y-axis, and it touches the parabola pow_2 in exactly one place. Follow these directions to determine the center and radius of the circle. First visualize the directions with rough sketches, and then use mathematical formalism to make your sketches precise.

(a) Take a to be a positive real number and determine the equation for a line tangent to pow_2 at (a, a^2).



The lowest point of a circle lies on the x-axis, the circle is to the right of the y-axis, and it touches the parabola pow_2 in exactly one place. Follow these directions to determine the center and radius of the circle. First visualize the directions with rough sketches, and then use mathematical formalism to make your sketches precise.

(b) A circle with center (b, c) touches pow_2 at (a, a^2). Determine the slope of the line tangent to the circle at (a, a^2).



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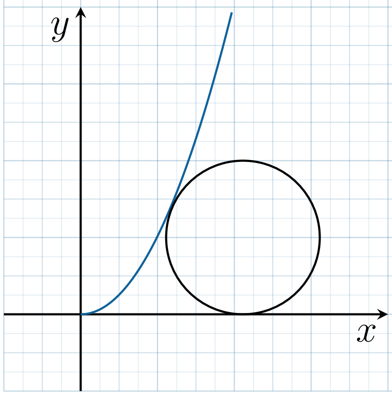
(c) Determine the relationship between the coordinates of the vector V that points from (b, c) to (a, a^2).



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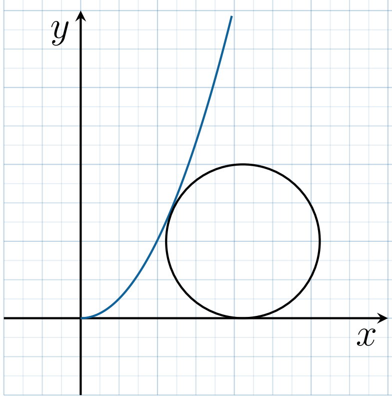
(d) Use part (c) to write b in terms of a and c.



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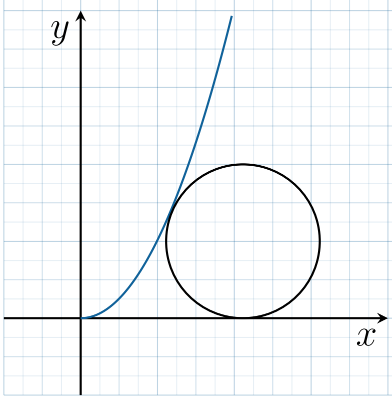
(e) The circle touches the x-axis. Determine what this implies about the radius of the circle and the length of the vector V.



The lowest point of a circle lies on the x-axis, the circle is to the right of the y-axis, and it touches the parabola pow_2 in exactly one place. Follow these directions to determine the center and radius of the circle. First visualize the directions with rough sketches, and then use mathematical formalism to make your sketches precise.

(f) Use part (e) to identify (b, c) for any a.

(g) Simulate the position of the point of contact (a, a^2), the circle with center (b, c), and the function pow_2.