Linguistic Mapping

The Principles of Calculus I

III

Rigidity

III.4

Orders of Intersection

Classroom Exercises

Exercise 1

The idea of a line being tangent at a point p to a special subset of $\mathbb{R}^2$ took a long time to precisely develop. Our presentation unfolds iteratively, with each step more nuanced that the previous. It is helpful to analyze the conceptual progression.

(1) For any quadratic polynomial f and any circle C, a line L_p is tangent to f or C at p if a certain condition holds on the number of intersections of L_p with f or C. Identify this condition.

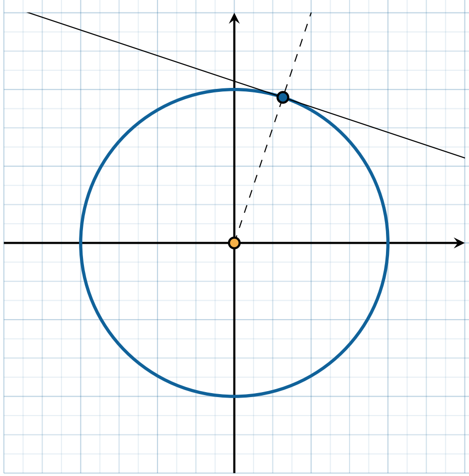
(2) Does this condition make sense for any polynomial f?

Exercise 2

Take C to be the circle of radius r centered at (a, b).

(a) Determine an equation for the line L_θ that is tangent to C at the point $r\langle\cos(\theta), \sin(\theta)\rangle + (a, b)$.

(b) Sketch C and L_θ using a computer and observe what happens as you change r, a, b, and θ.



Exercise 3

Take f to be the function given by

$$f(x) = x^2 - 2x - 5.$$

(a) Identify all lines ℓ_m that intersect f at $(2, -5)$, where m is the slope of ℓ_m.

(b) Identify the discriminant $D(m)$ of $f - \ell_m$.

Take f to be the function given by

$$f(x) = x^2 - 2x - 5.$$

(c) For what values of $D(m)$ will f and ℓ_m intersect at exactly one point?

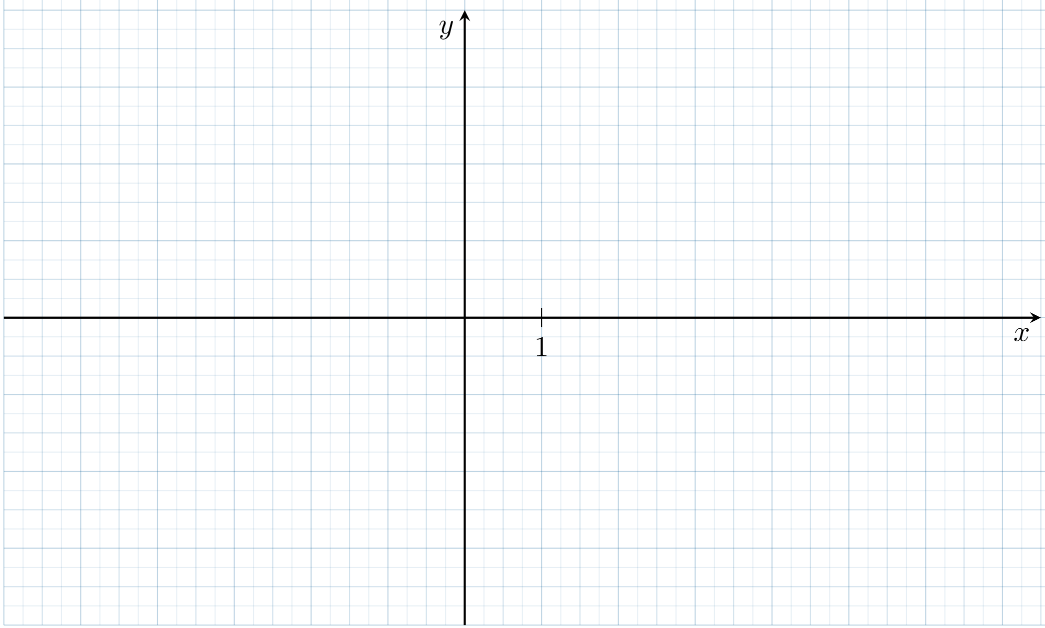
(d) Determine an equation of the line L_2 that is tangent to f at $(2, -5)$. Is this line unique?

Exercise 4

Take f to be the polynomial function that is given by

$$f(x) = (x+4)^2(x-1)(x-5)^3.$$

Sketch f and use the sketch to identify all points where the x-axis *should* be tangent to f.



Exercise 5

For any polynomial function f of degree 2 or greater, describe the following:

(a) What it means for a line L_a to have an order n intersection with f at $(a, f(a))$;

(b) How rewriting $f(x)$ as $f(a+(x-a))$ facilitates determining the order of intersection of f with L_a at $(a, f(a))$.

Exercise 6

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(3) For any polynomial f, a line L_a is tangent to f at $(a, f(a))$ if a certain condition holds on the order of intersection of f and L_a at $(a, f(a))$. Identify this condition.

Exercise 7

Take f to be the function given by

$$f(x) = x^2 - 2x - 5.$$

(a) Identify an equation for the line L that has a degree 2 intersection with f at $(2, -5)$.

(b) Determine an equation of the line L_2 that is tangent to f at $(2, -5)$.

Exercise 8

Take f to be the function given by

$$f(x) = x^4 + 3x^3 - 10x + 4.$$

(a) Identify an equation for the line L that has a degree 2 (or greater) intersection with f at $(1, -2)$.

(b) Determine an equation of the line L_1 that is tangent to f at $(1, -2)$.

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(4) For any rational function f, a line L_a is tangent to f at $(a, f(a))$ if a certain condition holds on the difference $f - L_a$. Identify this condition.

Exercise 10

Take f to be given by

$$f(x) = \frac{2x+1}{x-1}.$$

Determine an equation for the line tangent to f at $(4, 3)$.