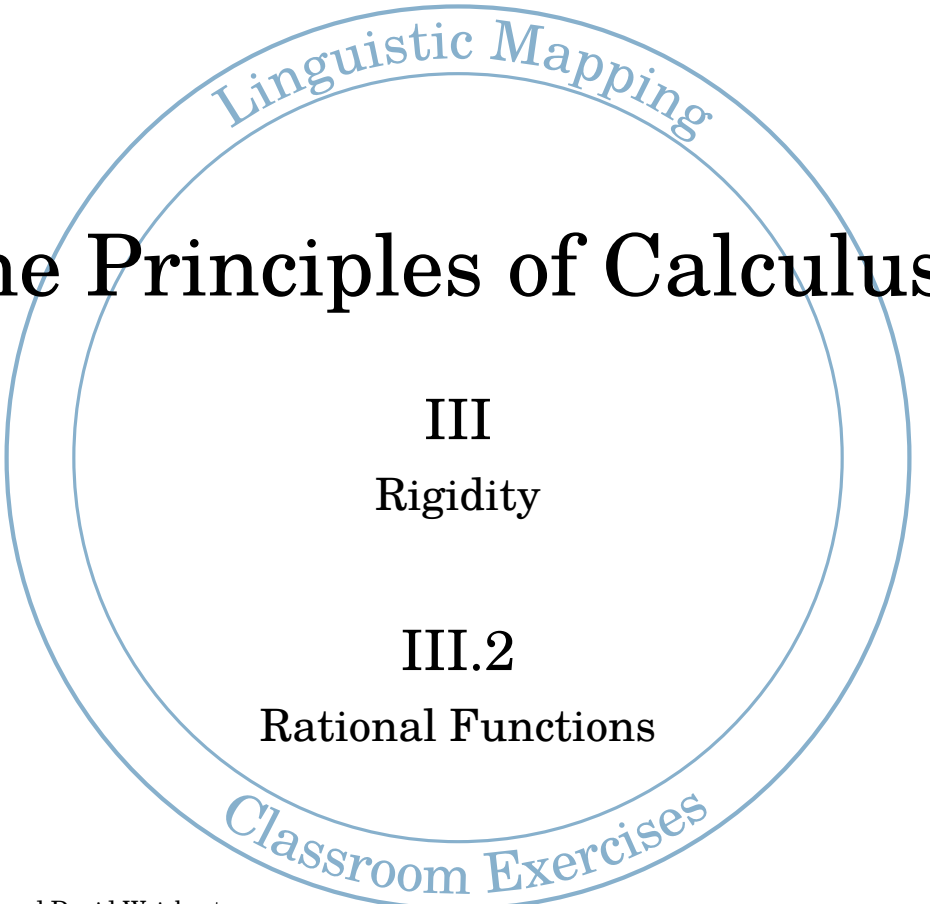

The Principles of Calculus I

III

Rigidity

III.2

Rational Functions

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Exercise 1

Explain what it means for a quotient of polynomial functions to be in simplest form and then identify each of these quotients that is given in simplest form:

(a) $\dfrac{(x+2)^5(x-5)}{(x+2)(x+3)^2}$; ☐

(b) $\dfrac{3x-1}{x}$; ☐

(c) $\dfrac{4(x+2)^2(x-5)}{2(x+1)(x-3)^3}$. ☐

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Exercise 2

Take f to be the rational function that is given by

$$f(x) = \frac{(x+5)(x-3)^2}{(x+4)^3(x-6)(x-7)^2}.$$

(a) Explain what it means for a rational function Q to have a pole of order m at a and a zero of order n at b.

(b) Identify all zeros and poles of f, and their orders.

Zero	Pole	Order

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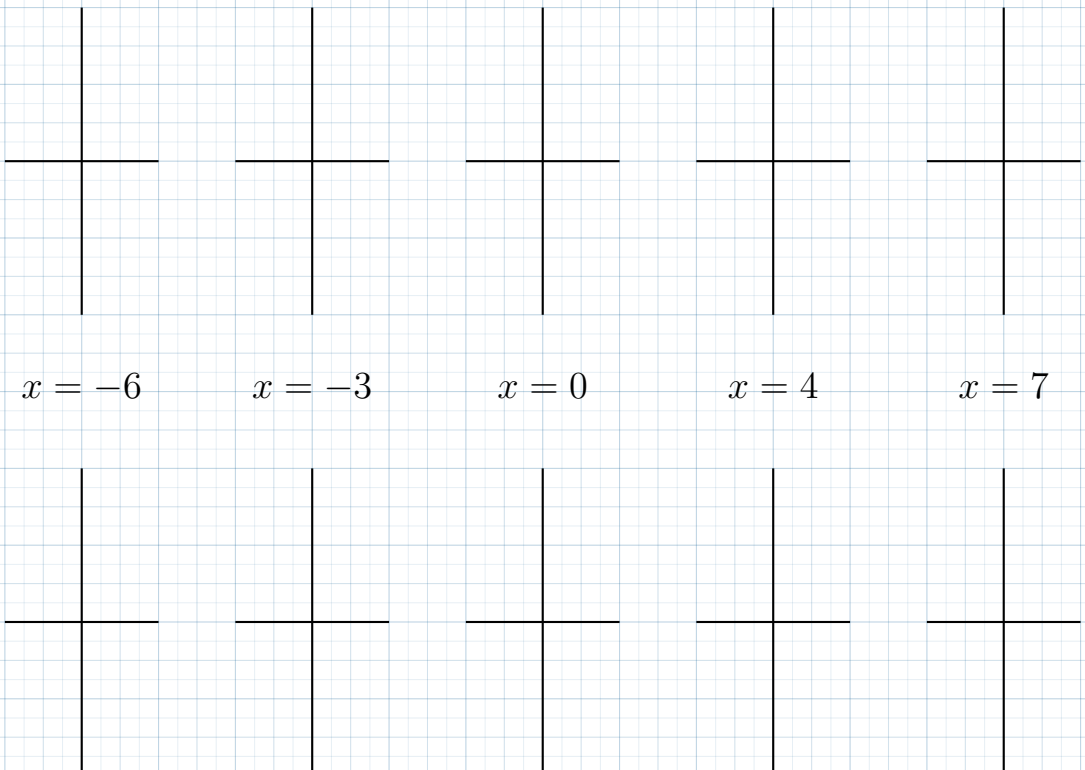
Exercise 3

A rational function f has a pole of order 3 at -3, a pole of order 2 at 7, a zero of order 1 at -6, a zero of order 2 at 0, and a zero of order 5 at 4. It may have additional zeros and poles. Sketch the possible local behavior of f.



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Exercise 4

For C a non-zero real number and n a natural number, explain what it means for a rational function f to be asymptotically equal to 0, C, or Cx^n, to be denoted

$$f \sim 0, \quad f \sim C, \quad \text{and} \quad f \sim Cx^n.$$

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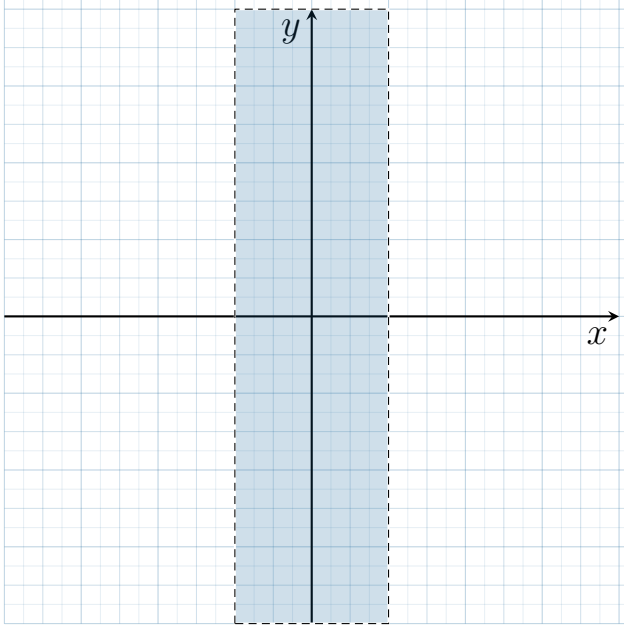
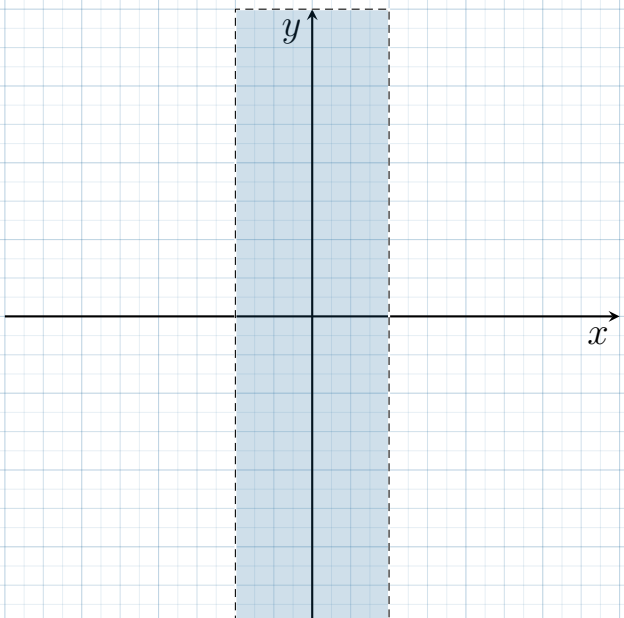
Exercise 5

Sketch f in a region far from the origin, for each of these choices of asymptotic behavior for f:

(a) $f \sim 0$;

(b) $f \sim 2$;





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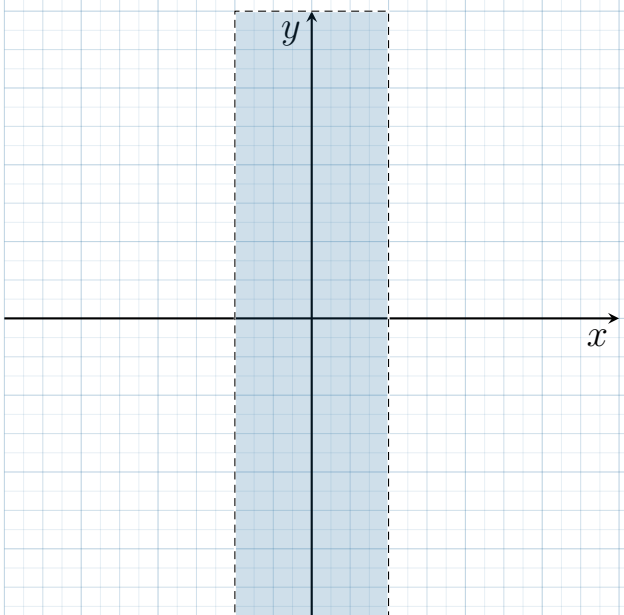
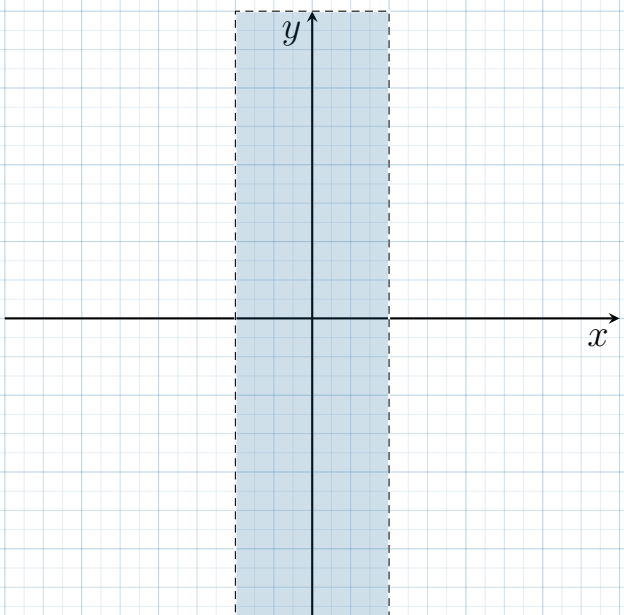
Sketch f in a region far from the origin, for each of these choices of asymptotic behavior for f:

(c) $f \sim 2x$;

(d) $f \sim -\frac{1}{3}x^2$;





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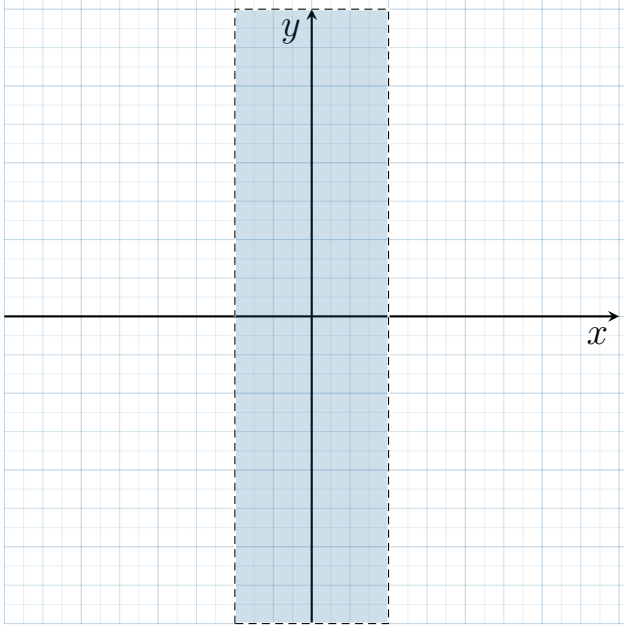
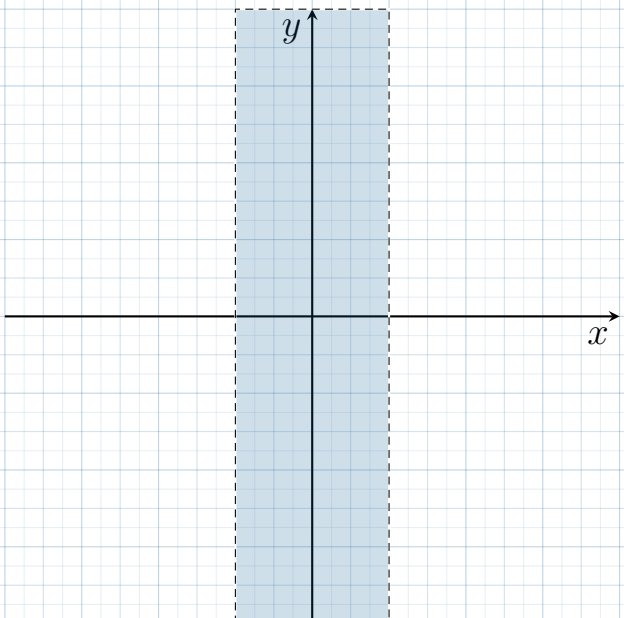
Sketch f in a region far from the origin, for each of these choices of asymptotic behavior for f:

(e) $f \sim -2x^3$;

(f) $f \sim x^4$.





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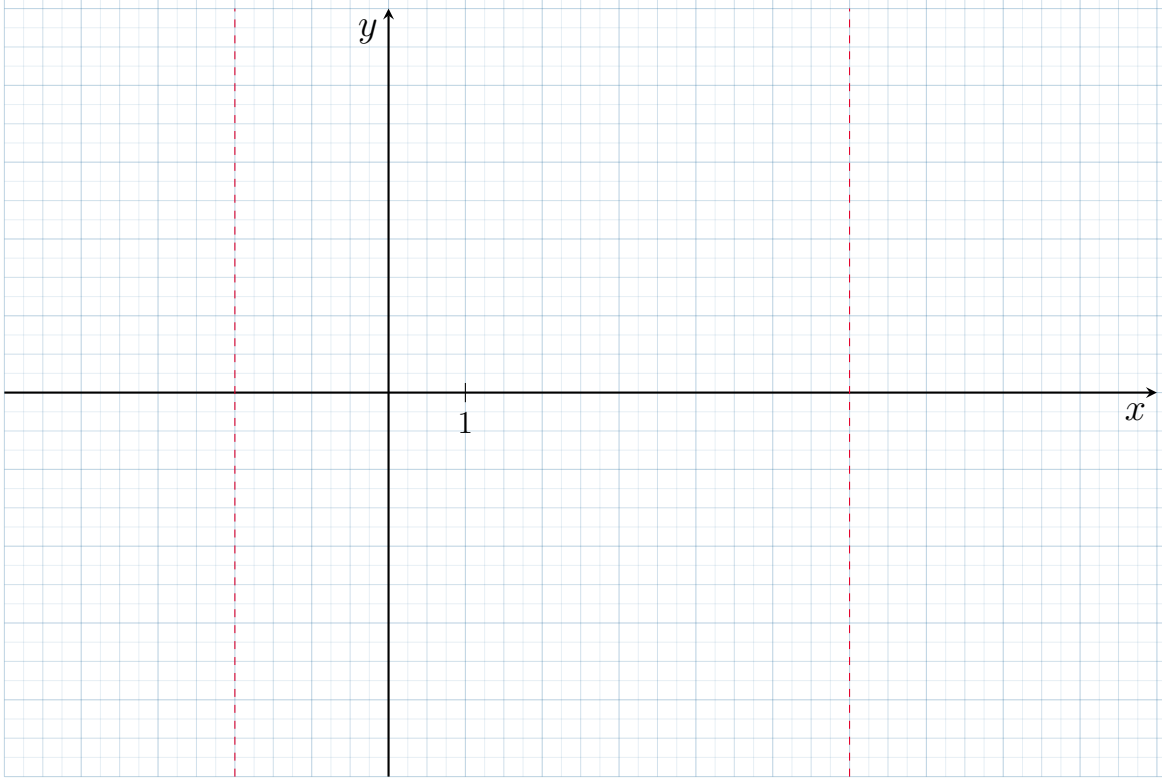
Exercise 6

Sketch f, where

$$f(x) = \frac{x-3}{(x+2)(x-6)}.$$



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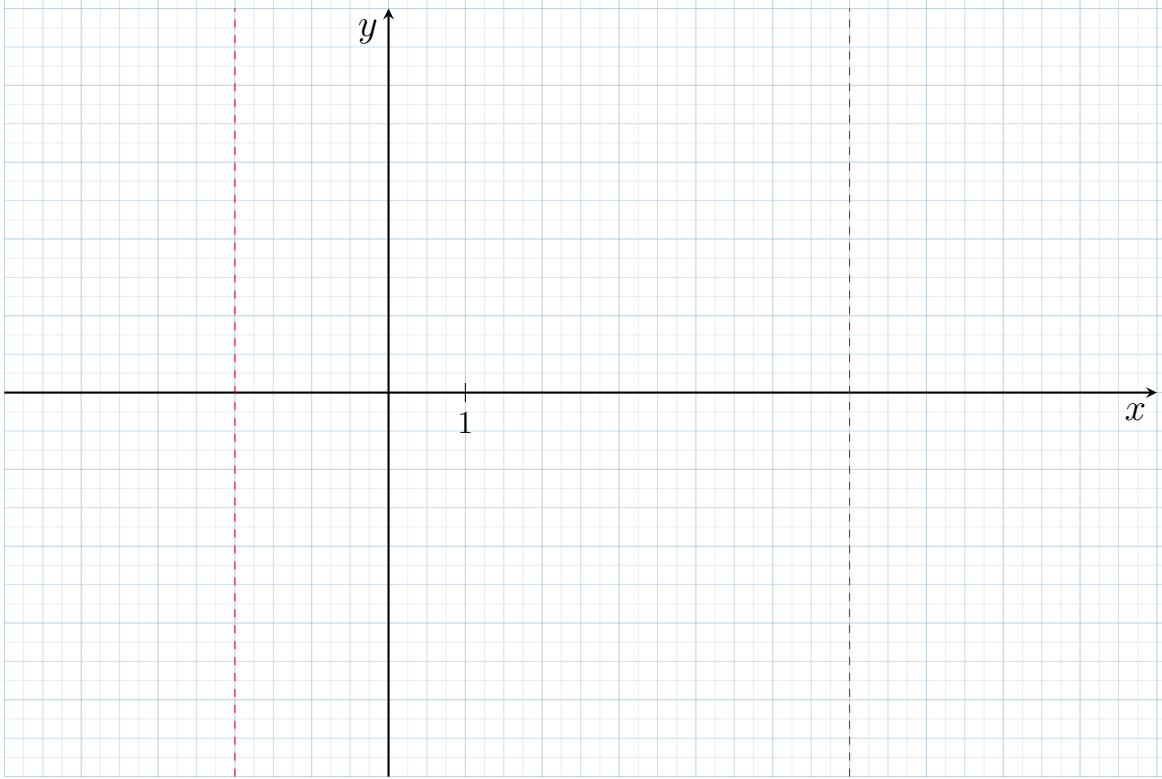
Exercise 7

Sketch f, where

$$f(x) = \frac{(x-3)^5}{(x+2)^2(x-6)^3}.$$



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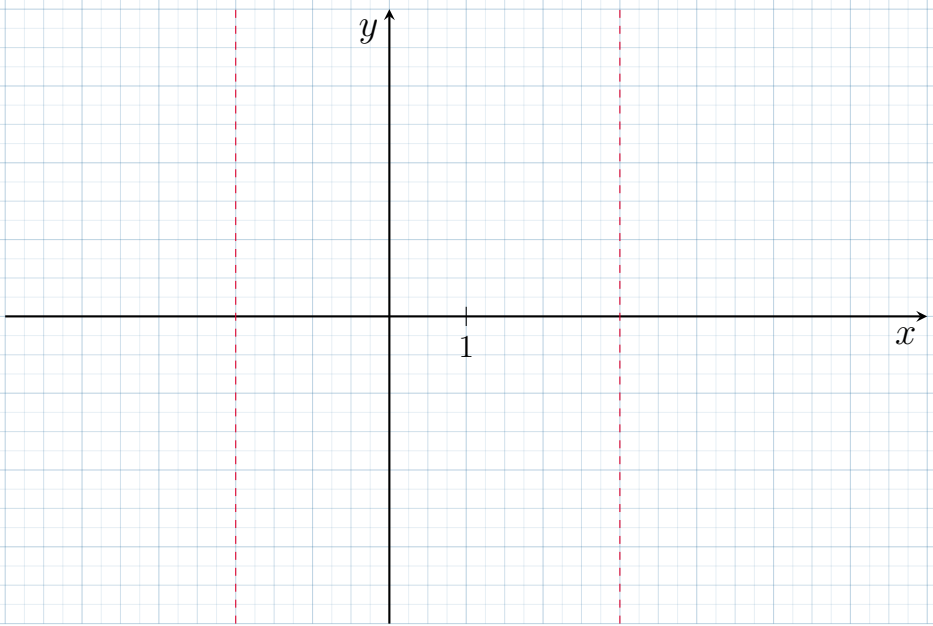
Exercise 8

For this choice of function f, sketch f first by hand and then with a computer, and study the behavior of the computer's sketch by scaling the x and y axes:

$$f(x) = \frac{(x+1)(x-1)^4(x-5)^3}{(x+2)^3(x-3)^2}.$$



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