

Linguistic Mapping

The Principles of Calculus I

II

Transformation

II.6

Describing Rotation in Cartesian Coordinates

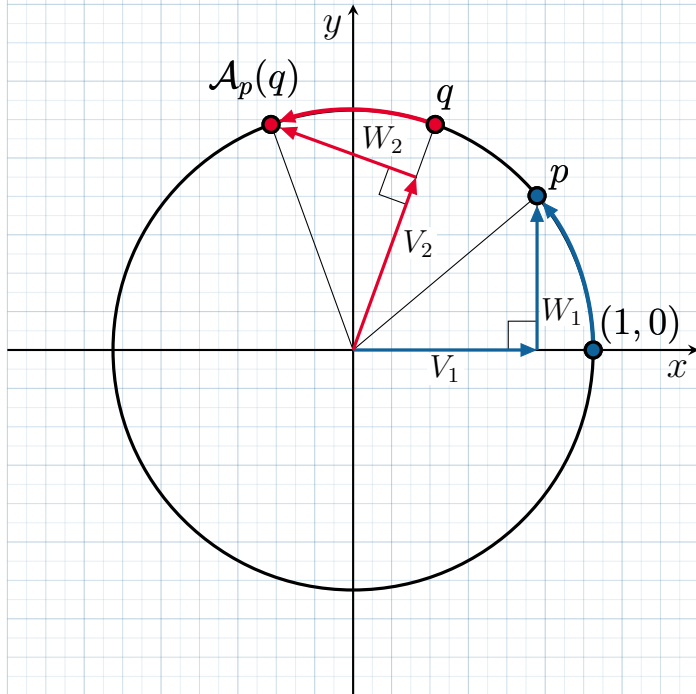
Classroom Exercises

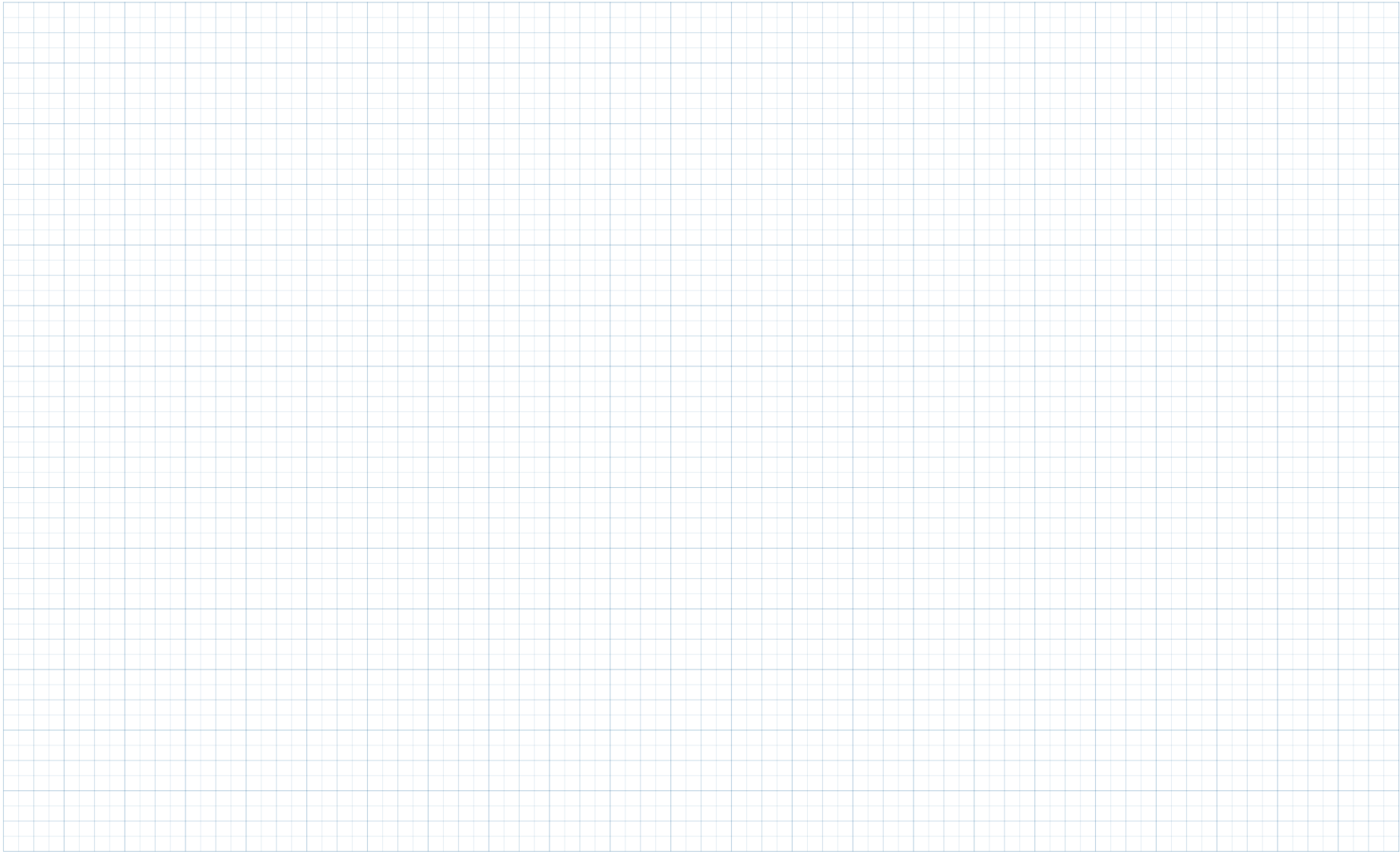
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Exercise 1

The arc \mathcal{A}_p from $(1, 0)$ to p on the unit circle below moves $(1, 0)$ to the point p . The motion is a rotation by the angle p . Given that p has coordinates (a, b) and q is a point on the unit circle with coordinates (c, d) , determine how the arc \mathcal{A}_p acts to move q to the point that we will denote by $\mathcal{A}_p(q)$.





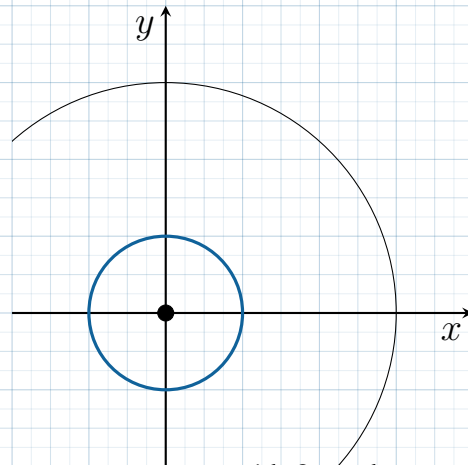
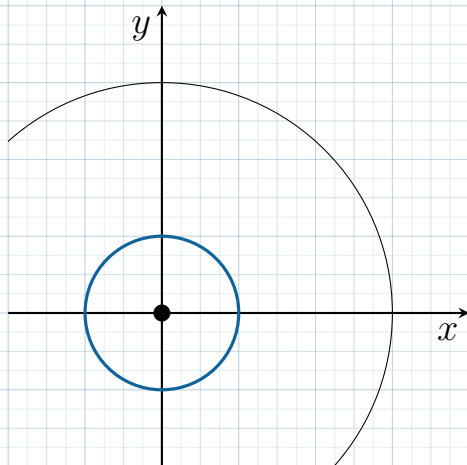
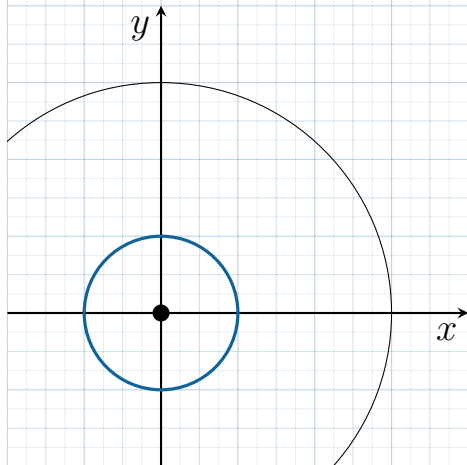
Exercise 2

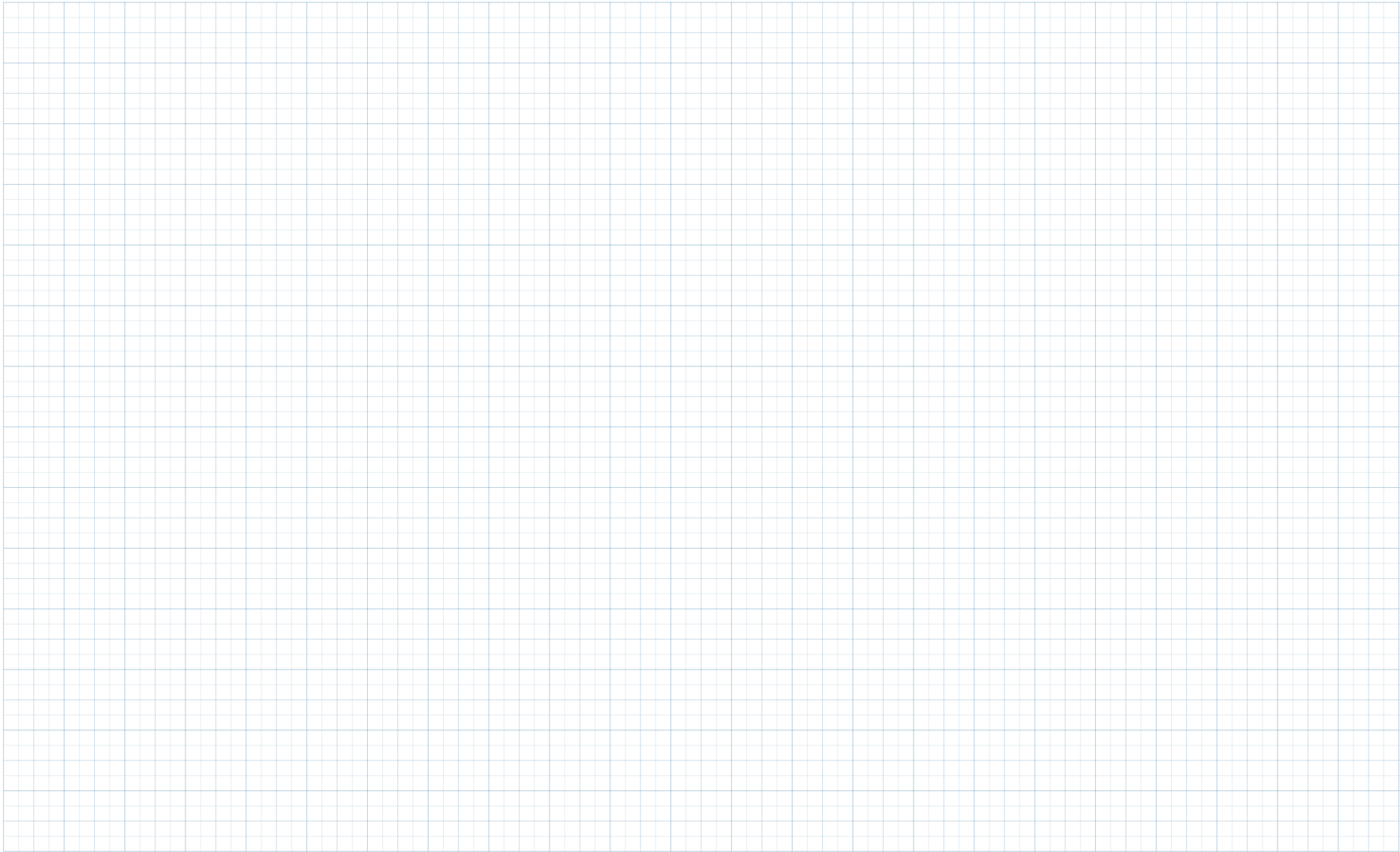
Take (a, b) and (c, d) to be points on the unit circle, (x, y) to be a point in \mathbb{R}^2 , and $\langle x, y \rangle$ to be a vector. Describe the meaning of these quantities in plain English and with a clarifying picture, and precisely compute each:

(a) $(a, b) \star (c, d) =$;

(b) $(a, b) \star (x, y) =$;

(c) $(a, b) \star \langle x, y \rangle =$.





Exercise 3

For any real number a in $[-1, 1]$, rotate the point $(3, 4)$ around the point $(2, 1)$ by the angle $(a, \sqrt{1 - a^2})$. Simulate how this rotation moves $(3, 4)$ for different values of a . Performing this operation amounts to translating an English sentence into a precise mathematical statement. This is the statement:

Rotate the difference between $(3, 4)$ and $(2, 1)$ by the angle $(a, \sqrt{1 - a^2})$ and add the resulting vector to $(2, 1)$.

