Linguistic Mapping

The Principles of Calculus I

II

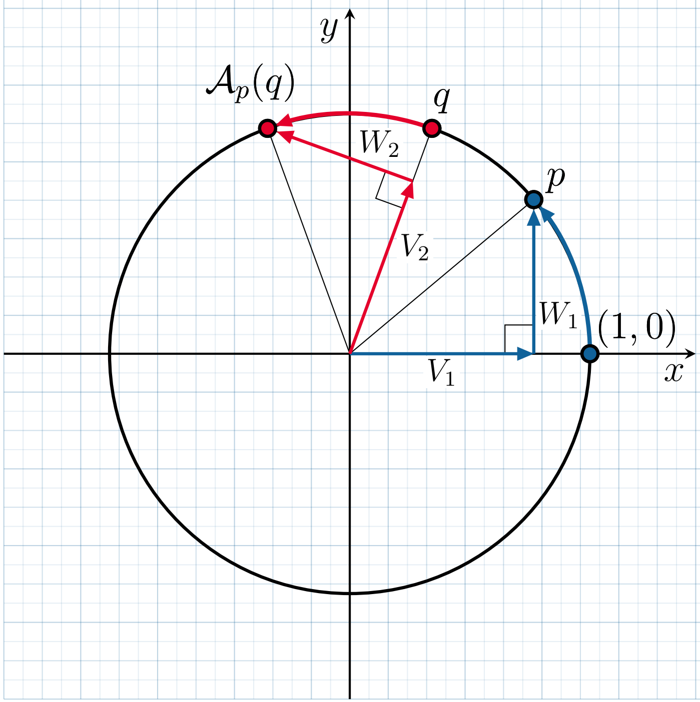
Transformation

II.6

Describing Rotation in Cartesian Coordinates

Classroom Exercises

The arc $\mathcal{A}_p$ from $(1,0)$ to p on the unit circle below moves $(1,0)$ to the point p. The motion is a rotation by the angle p. Given that p has coordinates (a,b) and q is a point on the unit circle with coordinates (c,d), determine how the arc $\mathcal{A}_p$ acts to move q to the point that we will denote by $\mathcal{A}_p(q)$.



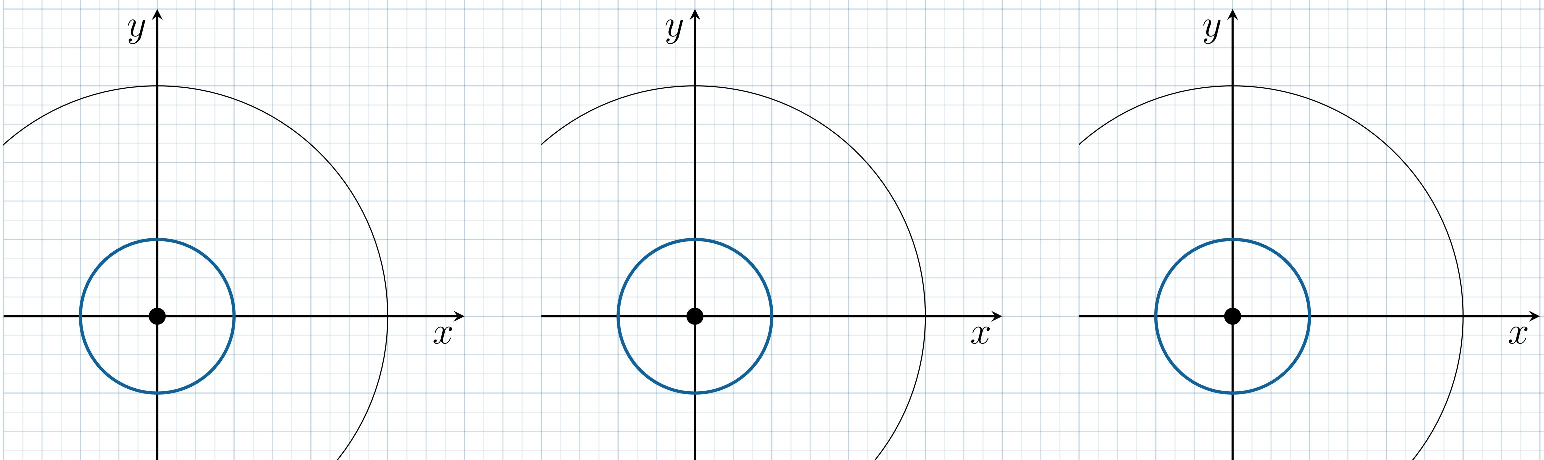
Exercise 2

Take (a, b) and (c, d) to be points on the unit circle, (x, y) to be a point in $\mathbb{R}^2$, and $\langle x, y \rangle$ to be a vector. Describe the meaning of these quantities in plain English and with a clarifying picture, and precisely compute each:

(a) $(a, b) \star (c, d) =$ __________ ;

(b) $(a, b) \star (x, y) =$ __________ ;

(c) $(a, b) \star \langle x, y \rangle =$ __________ .



Exercise 3

For any real number a in $[-1, 1]$, rotate the point $(3, 4)$ around the point $(2, 1)$ by the angle $\left(a, \sqrt{1-a^2}\right)$. Simulate how this rotation moves $(3, 4)$ for different values of a. Performing this operation amounts to translating an English sentence into a precise mathematical statement. This is the statement:

Rotate the difference between $(3, 4)$ and $(2, 1)$ by the angle $\left(a, \sqrt{1-a^2}\right)$ and add the resulting vector to $(2, 1)$.