Linguistic Mapping

The Principles of Calculus I

II

Transformation

II.5

Inverse Functions

Classroom Exercises

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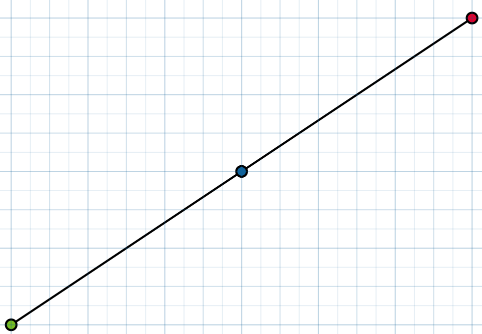
Exercise 1

Determining the midpoint m of a line segment L with endpoints (a, b) and (c, d) comes down to translating an intuitive statement in English to formal mathematics!

To see this, translate this statement into mathematical symbols: Add half the difference between (c, d) and (a, b) to move (a, b) to the midpoint m of L.



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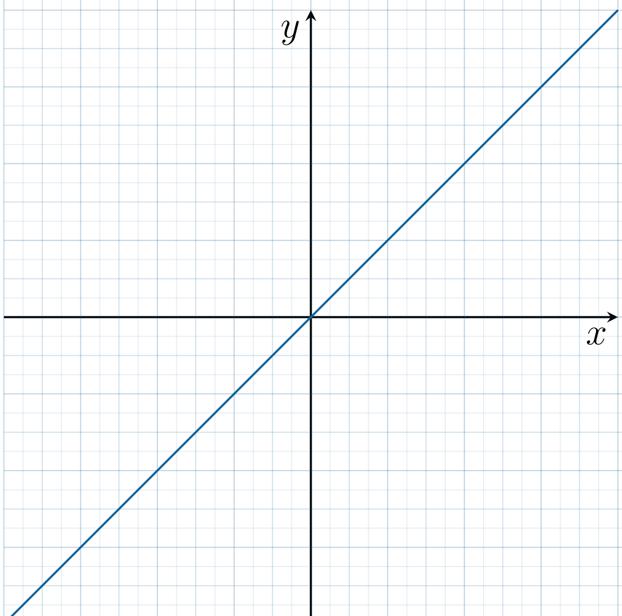
Exercise 2

For any line L, denote by Ref_L the reflection across L. Denote by Ref_1 rather than $\mathrm{Ref}_{\mathrm{pow}_1}$ the reflection across pow_1 to simplify typography. Sketch the line pow_1 and for any real number a the points $(a, 0)$ and $(0, a)$. How do you think that $(a, 0)$ and $(0, a)$ should reflect across pow_1?



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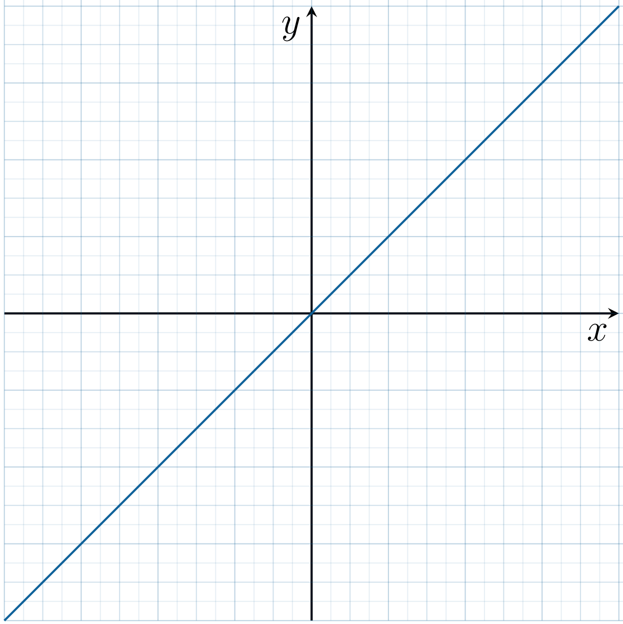
Exercise 3

For any nonzero real number a, determine the slope of the line $L_\perp$ that contains $(a, 0)$ and $(0, a)$ and determine the midpoint of the line segment with endpoints $(a, 0)$ and $(0, a)$. Use these values to argue that

$$\mathrm{Ref}_1(a, 0) = (0, a) \quad \textbf{and} \quad \mathrm{Ref}_1(0, a) = (a, 0).$$



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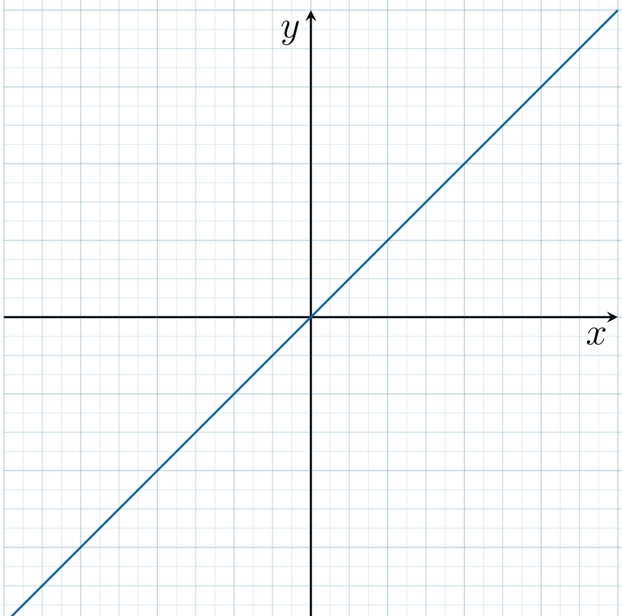
Determine the midpoint m of the line segment $L_\perp$ with endpoints (a,b) and (b,a) as well as the slope of $L_\perp$. Conclude that

$$\mathrm{Ref}_1(a,b) = (b,a).$$



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Exercise 5

For any function f that is a bijection from $\mathcal{D}(f)$ to $\mathcal{R}(f)$, $\mathrm{Ref}_1(f)$ is a function from $\mathcal{R}(f)$ to $\mathcal{D}(f)$. Denote by f^{-1} the function $\mathrm{Ref}_1(f)$.

(a) A line L contains the point $(1, 3)$ and has slope 5. Given any distinct points (a, b) and (c, d) in L, if L has slope 5, then

$$\frac{d-b}{c-a} = \boxed{}.$$

(b) Compute $\mathrm{Ref}_1(a, b)$ and $\mathrm{Ref}_1(c, d)$ to determine the slope of L^{-1}.

(c) Determine an equation for L^{-1}.

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Exercise 6

Determining a formula for an inverse function amounts to changing the words one uses to label the components of a function. As an example, take f to be the function that is given by

$$f(x) = \frac{3x-1}{x+2}.$$

(a) Describe f using set builder notation and compute $\mathrm{Ref}_1(f)$.

(b) Rewrite $\mathrm{Ref}_1(f)$ so that the first component is given by an independent variable and the second component depends on the first.

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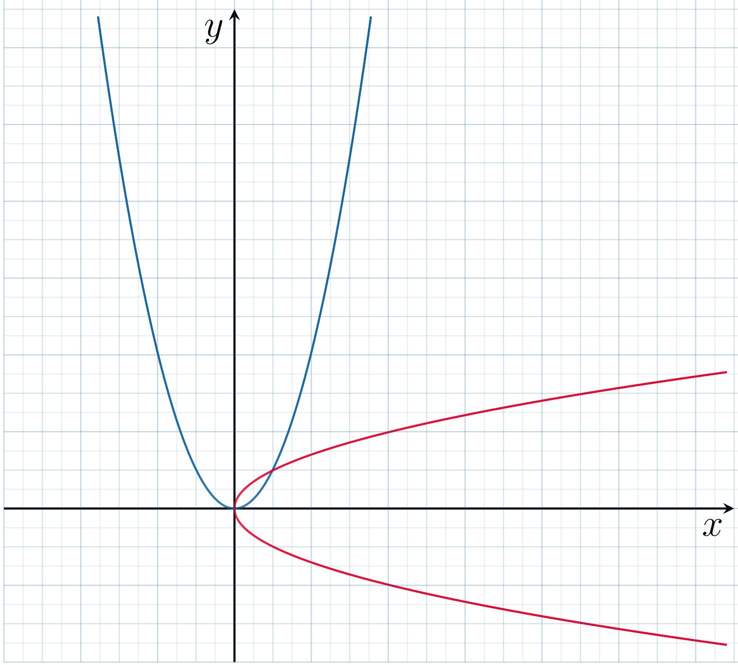
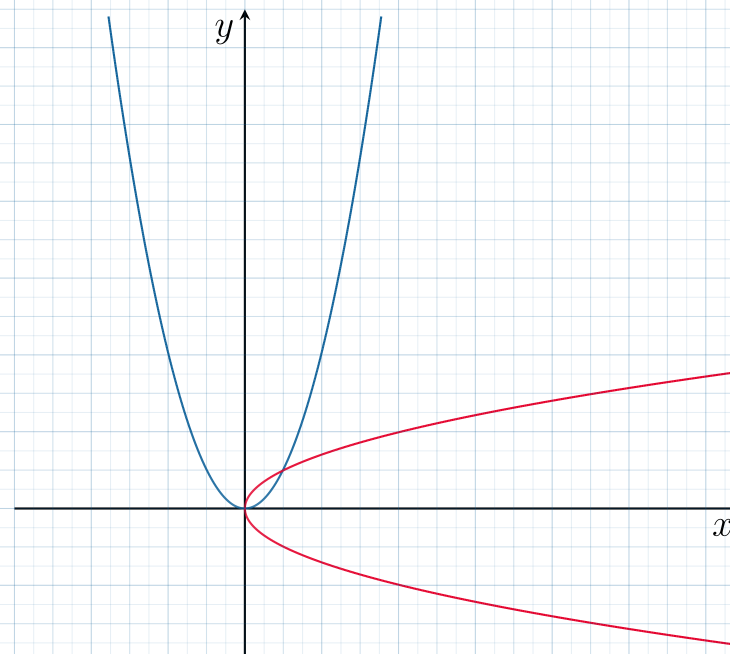
The function pow_2 is not invertible.

(a) Do you see why pow_2 is not invertible on $\mathbb{R}^2$? What does $\mathrm{Ref}_1(\mathrm{pow}_2)$ look like?

(b) Formulate a precise argument to show that pow_2 is not invertible.





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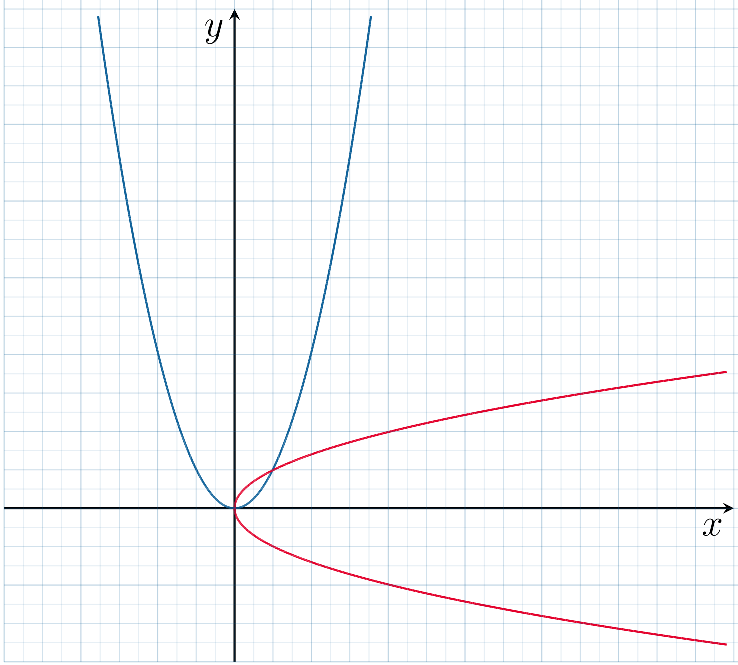
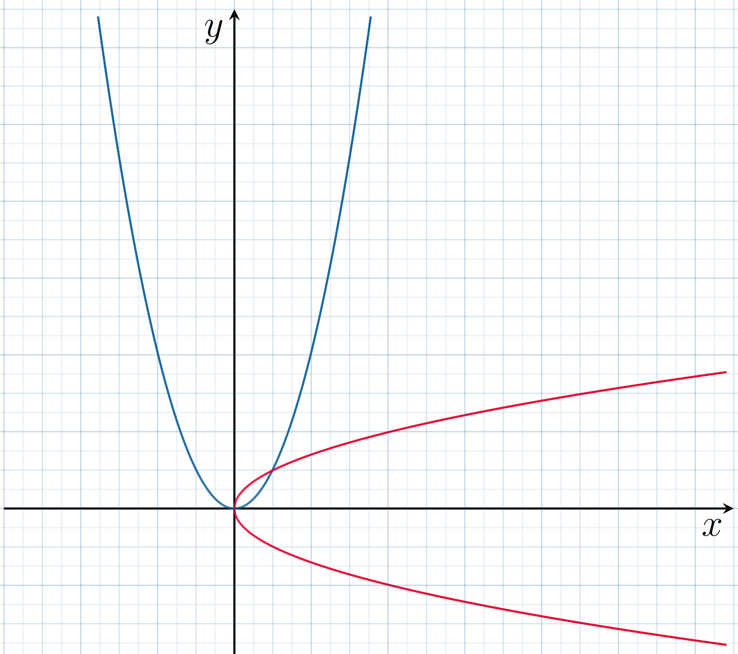
Take S to be the union of intervals

$$S = (-3, -2] \cup [0, 2) \cup [3, \infty).$$

(a) Do you see why pow_2 is invertible on S? What does $\mathrm{Ref}_1(\mathrm{pow}_2)$ look like?

(b) Without making a single calculation, present $\left(\mathrm{pow}_2 \big|_S \right)^{-1}$ as a piecewise function.





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Exercise 9

Take S to be the union of intervals

$$S = (-3, -2] \cup [0, 2) \cup [3, \infty).$$

(a) Identify a partition P for S with as few intervals as possible.

(b) For each interval I in P, determine the range of $\mathrm{pow}_2\big|_I$ and a formula for $\left(\mathrm{pow}_2\big|_I\right)^{-1}$.

(c) Identify a partition P' for the domain of $\left(\mathrm{pow}_2\big|_I\right)^{-1}$ so that $\left(\mathrm{pow}_2\big|_I\right)^{-1}$ is given by a single formula on each interval in P'.

(d) Present $\left(\mathrm{pow}_2\big|_S\right)^{-1}$ as a piecewise defined function with respect to P'.

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