

Linguistic Mapping

The Principles of Calculus I

II

Transformation

II.4

Orthogonality and Reflection

Classroom Exercises

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Exercise 1

Take V to be a vector that moves points in the plane.

- (a) How do you read the symbol V_{\perp} ?
- (b) Describe in plain English the meaning of the symbol V_{\perp} and its relationship to V .
- (c) Given that V is equal to $\langle a, b \rangle$, express V_{\perp} in coordinates.

Exercise 2

Take L to be the line that is given by the equation

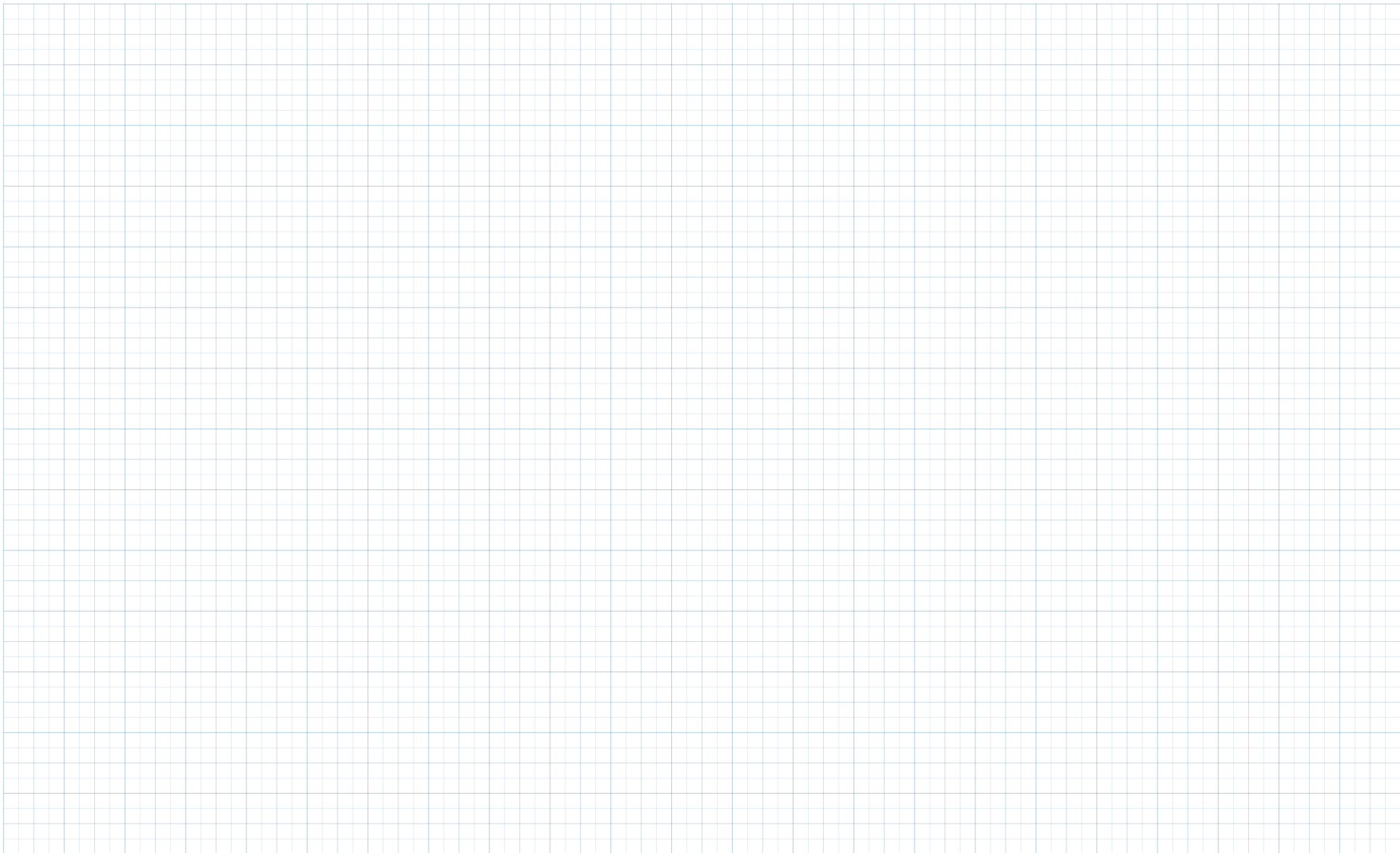
$$y = 3x - 2.$$

- (a) Identify a vector V that moves points along L .
- (b) Determine the vector V_{\perp} .
- (c) Identify an equation for the line that is perpendicular to L and that contains the point $(2, -1)$.

Exercise 3

The lower right vertex of a rectangle R has coordinates $(2, 1)$ and the upper right vertex has coordinates $(3, 7)$. The short side of the rectangle has length 2.

- (a) Sketch the rectangle R .
- (b) Label the unknown vertices as p and q , where p is the upper left vertex and q is the lower left vertex, and take V to be the vector $(3, 7) - (2, 1)$.
- (c) The vectors $p - (3, 7)$ and $q - (2, 1)$ both have polar form .
- (d) Use (c) to precisely determine p and q .

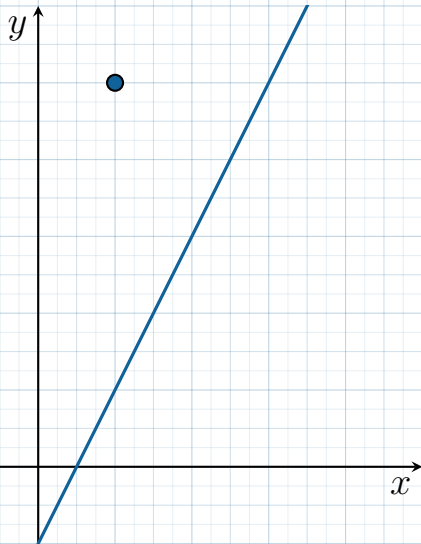


Exercise 4

Take L to be the line given by

$$y = 2x - 1.$$

- (a) In order to find the point on L that is closest to $(1, 5)$, first assume that there is such a point, call it p , and take its coordinates to be (x_0, y_0) . Roughly sketch p .
- (b) Draw a line L_\perp from $(1, 5)$ to p . What is the relationship between a unit vector V that moves points along L , and a unit vector W that moves points along L_\perp ?

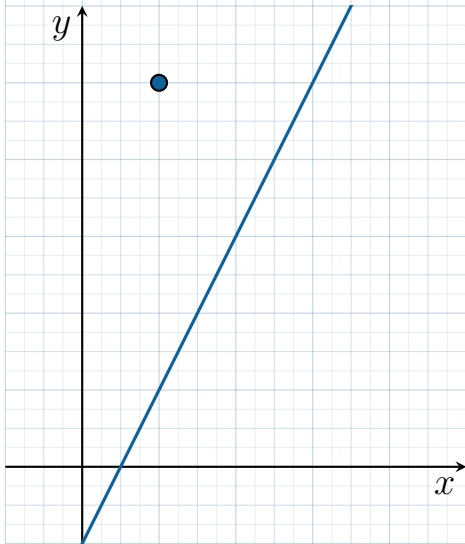


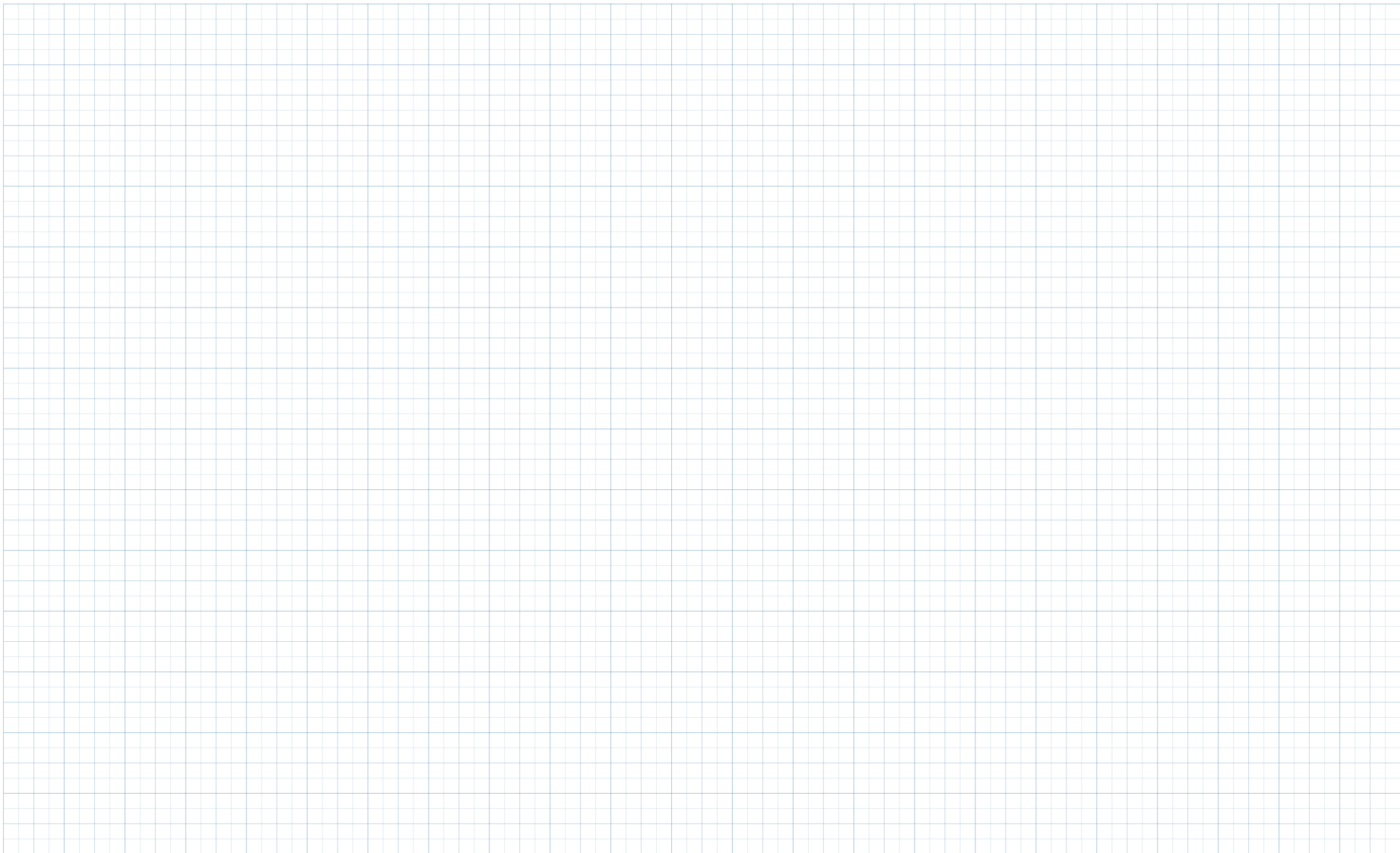
Exercise 5

Take L to be the line given by

$$y = 2x - 1.$$

- (a) Identify an equation for the line L_{\perp} that contains $(1, 5)$ and p , where p is the point in L that is closest to $(1, 5)$. Determine the slope of L_{\perp} .
- (b) The point p is the point at which L_{\perp} and L intersect. Precisely determine p .





Exercise 6

Take L to be the line given by

$$y = 2x - 1.$$

- (a) Take q to be the reflection of the point $(1, 5)$ across the line L . Roughly sketch q .
- (b) Where does the line that contains $(1, 5)$ and q intersect L ?
- (c) Precisely determine q .

