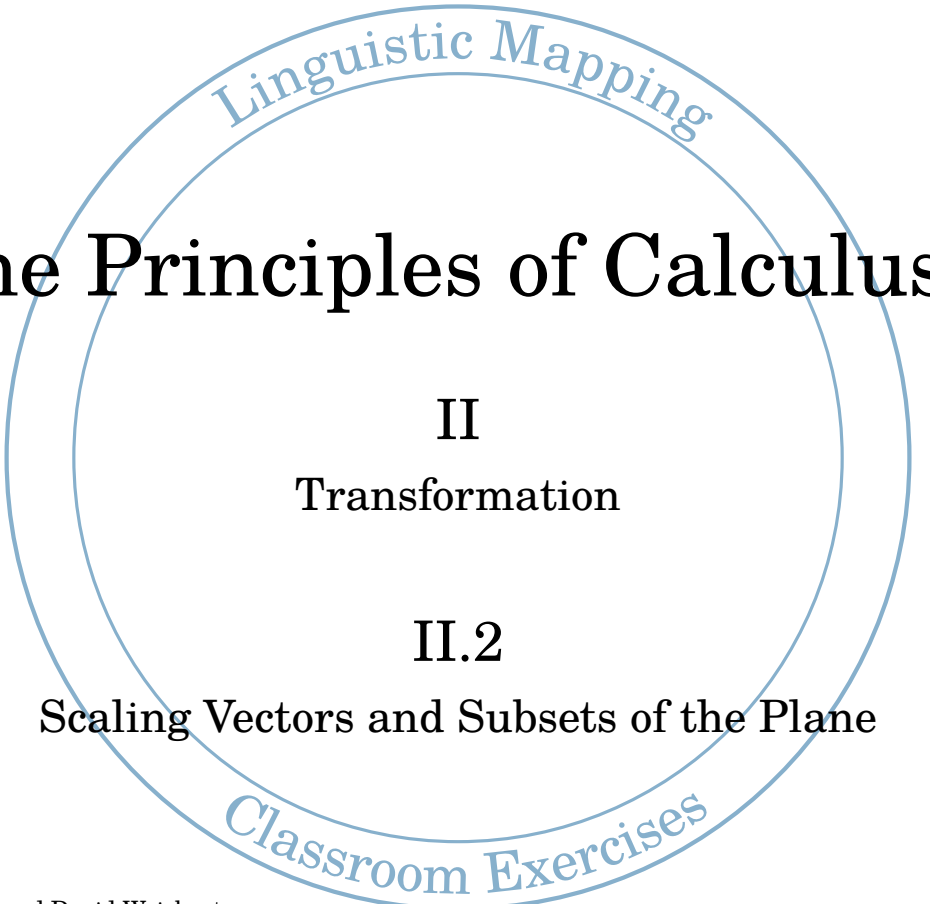


The Principles of Calculus I

II

Transformation

II.2

Scaling Vectors and Subsets of the Plane

Exercise 1

Compute the following quantities and explain in plain English the meaning of the algebraic operations that involve the vectors:

(a) $3\langle 2\rangle =$ $\square$

(b) $-5\langle 2\rangle =$ $\square$

(c) $2\langle 3\rangle + \langle 4\rangle =$ $\square$

(d) $3\langle 2, -3\rangle =$ $\square$

(e) $-2\langle 1, 3\rangle + \langle 2, 6\rangle =$ $\square$

Exercise 2

The vector V moves the point $(1, 4)$ to the point $(3, 7)$.

(a) How do you read the notation $\|V\|$, and what does this symbol mean?

(b) How do you read the notation $\hat{V}$, and what does this symbol mean?

Exercise 3

The vector V moves the point $(1, 4)$ to the point $(3, 7)$.

(a) Write V in polar form. What information about V does its expression in polar form immediately reveal?

(b) Identify all points p that are a distance of 3 away from $(1, 4)$ in the direction of V. Note: In the direction of V could mean both to the left and to the right of $(1, 4)$.

Exercise 4

Determine an equation for the set $\mathscr{C}_5(1,2)$ that consists of all points in $\mathbb{R}^2$ that are a distance of 5 from $(1,2)$. Use set builder notation to describe $\mathscr{C}_5(1,2)$.

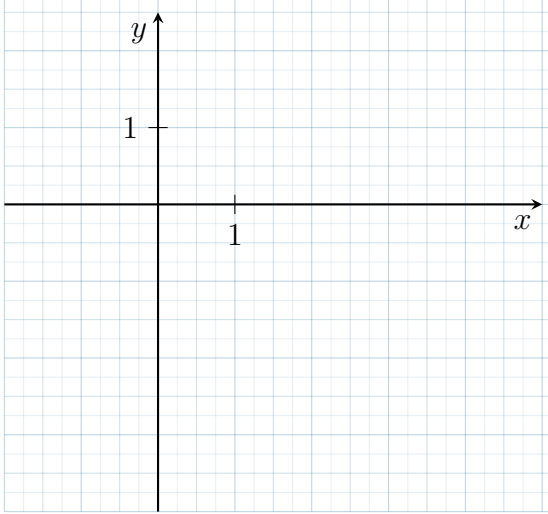
Exercise 5

(a) Sketch the circle $\mathscr{C}_2(2,-1)$ of radius 2 that is with center equal to $(2,-1)$ and sketch the point $(4,2)$.

(b) Sketch the vector V that moves $(2,-1)$ to $(4,2)$. How does the formal expression of this vector as a difference of points help you to determine its coordinates?

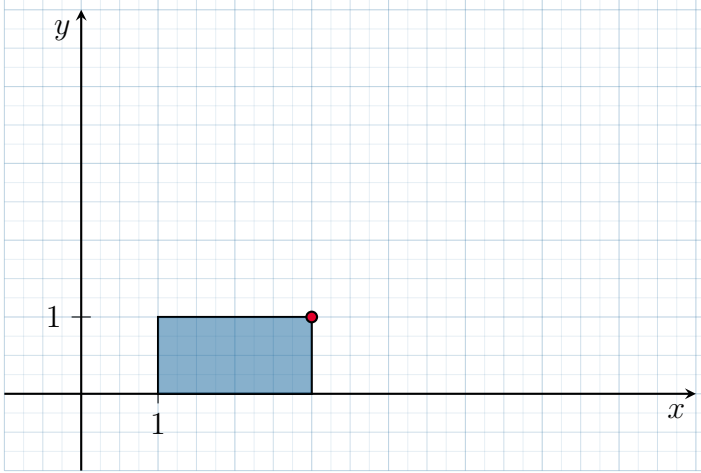
(c) Write V in polar form. What information does this form reveal about the two points?

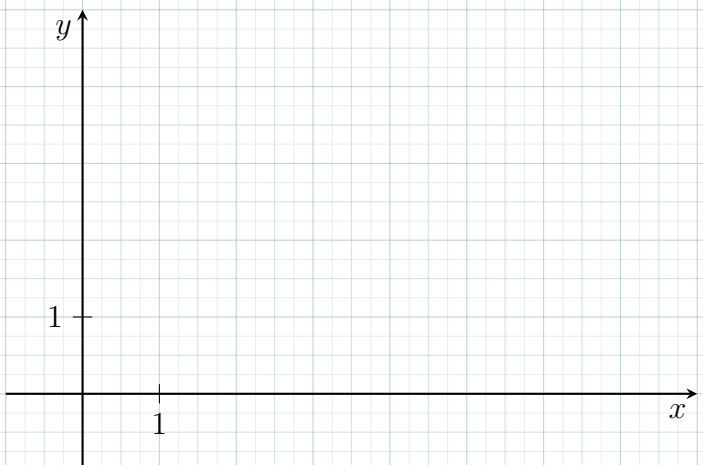
(d) Determine the projection of $(4,2)$ onto $\mathscr{C}_2(2,-1)$.

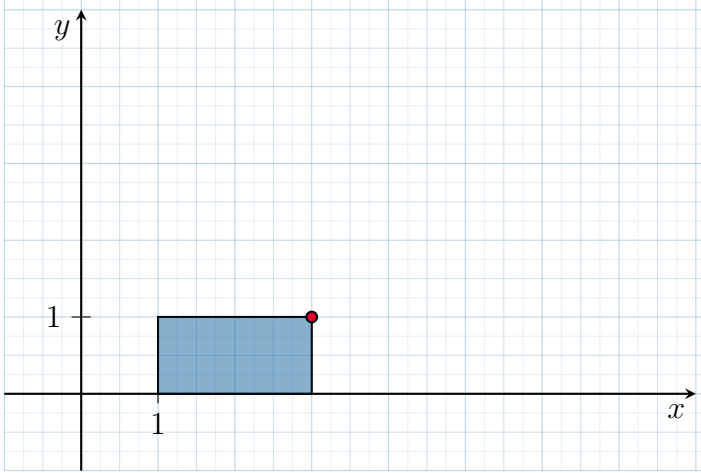


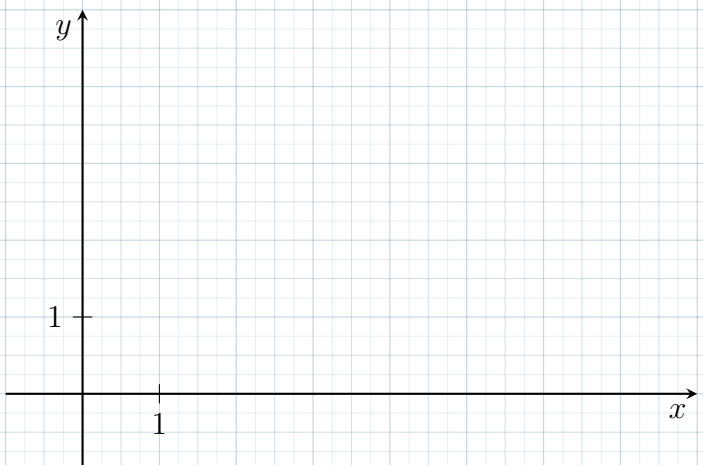
Exercise 6

Take X_2 and Y_4 to be the asymmetric scalings of the x-axis and y-axis, respectively. Describe what the symbols $X_2(3,1)$ and $Y_4(3,1)$ mean. It may be helpful to make a sketch and transform the rectangle that is drawn along with the point $(3,1)$.









Exercise 7

Use the scalings X_2 and Y_4 to asymmetrically scale the set $\mathscr{C}_5(1,2)$ that consists of all points in $\mathbb{R}^2$ that are a distance of 5 from $(1,2)$.

(a) First, describe $\mathscr{C}_5(1,2)$ using set builder notation.

(b) Second, explain what the symbol $Y_4X_2(\mathscr{C}_5(1,2))$ means.

(c) Third, use set builder notation to describe set $E = Y_4X_2(\mathscr{C}_5(1,2))$.

(d) Finally, use a change of variables to identify a formula that describes E. Be sure to use a graphing application to sketch E.