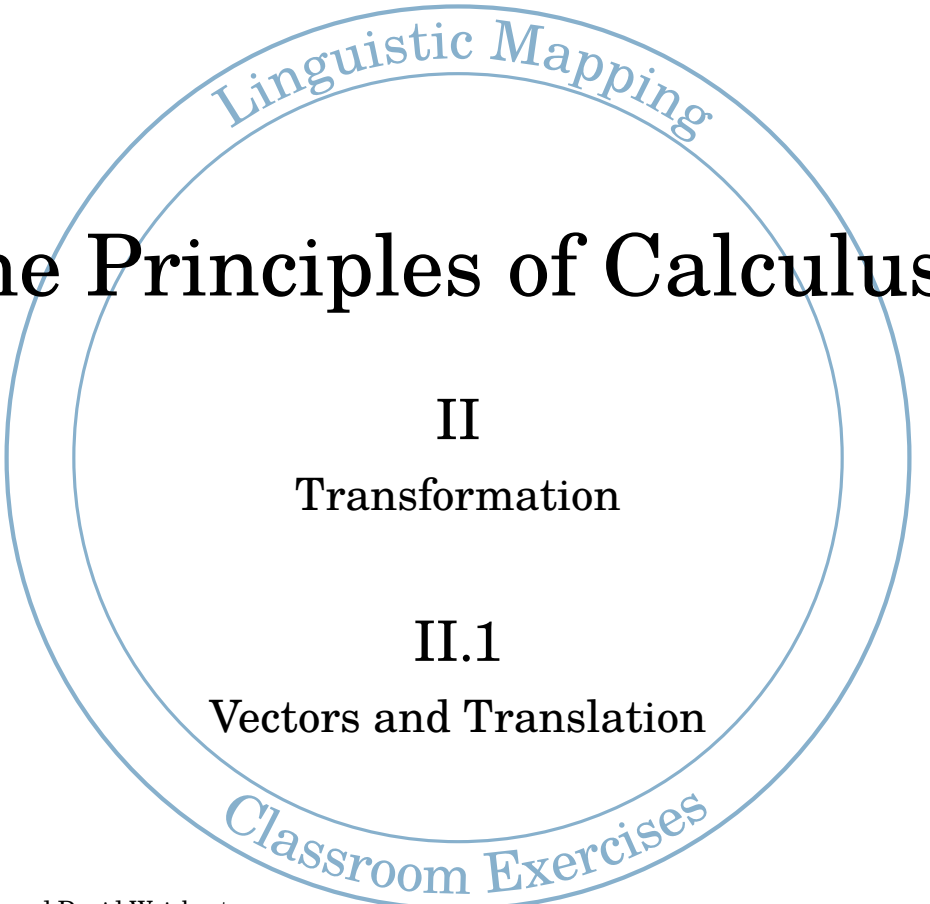


The Principles of Calculus I

II

Transformation

II.1

Vectors and Translation

Exercise 1

(a) Sketch the points 3, 5, 6, and 8 on the real number line.

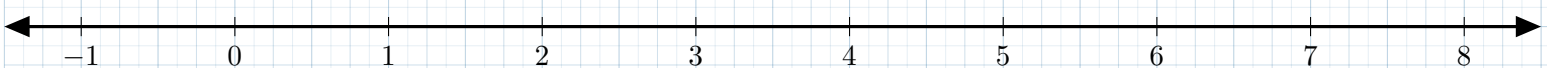
(b) Draw arrows from 3 to 5 and from 6 to 8.

The arrows represent (the vector) $\langle 2 \rangle$ and their placement represents how $\langle 2 \rangle$ moves 3 to 5 and 6 to 8. Write

$$\langle 2 \rangle + 3 = 5$$

to capture the meaning that $\langle 2 \rangle$ moves 3 to 5.

(c) Write in words how you would refer to the symbol $\langle 2 \rangle$ in spoken language.



Exercise 2

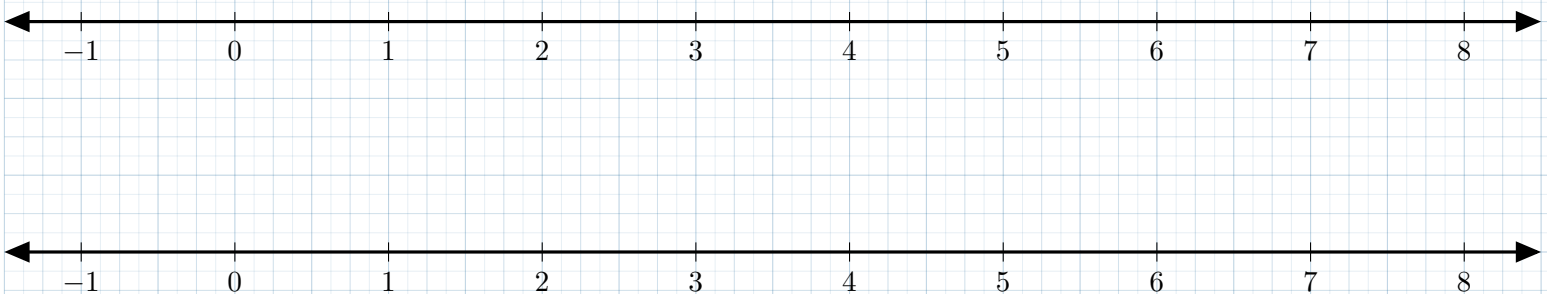
(a) Sketch the points 2 and 5 on the real number line.

(b) Define $5 - 2$ to be the vector that moves 2 to 5 and represent this as an arrow on the number line below.

(c) Write the vector $5 - 2$ and the vector $6 - 3$ using the notation $\langle \cdot \rangle$. What physical meaning does the similarity of the representation of the two quantities in the given notation capture?

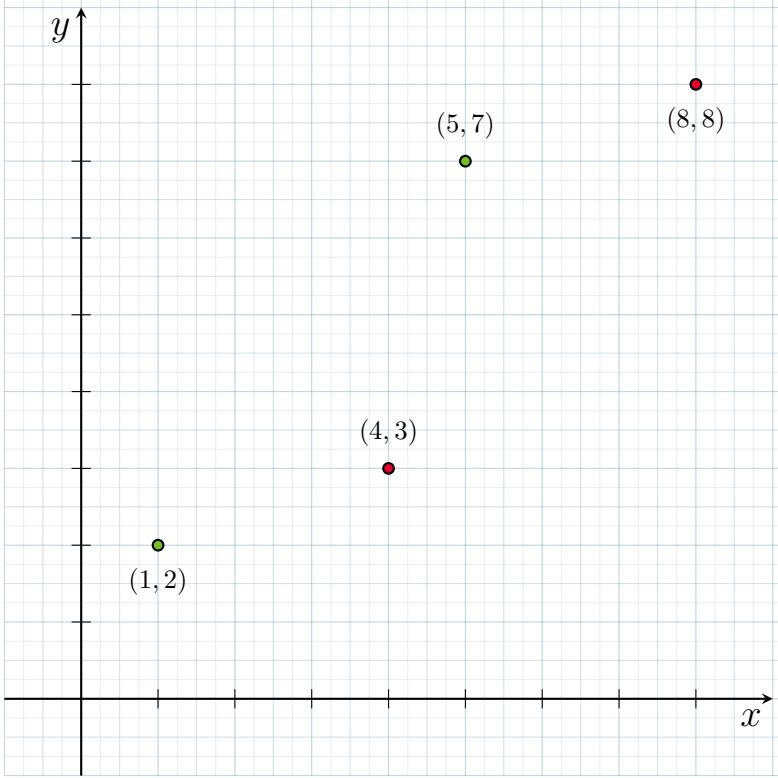
(d) Draw a picture that represents the equation

$$(5 - 2) + 4 = 7.$$



Exercise 3

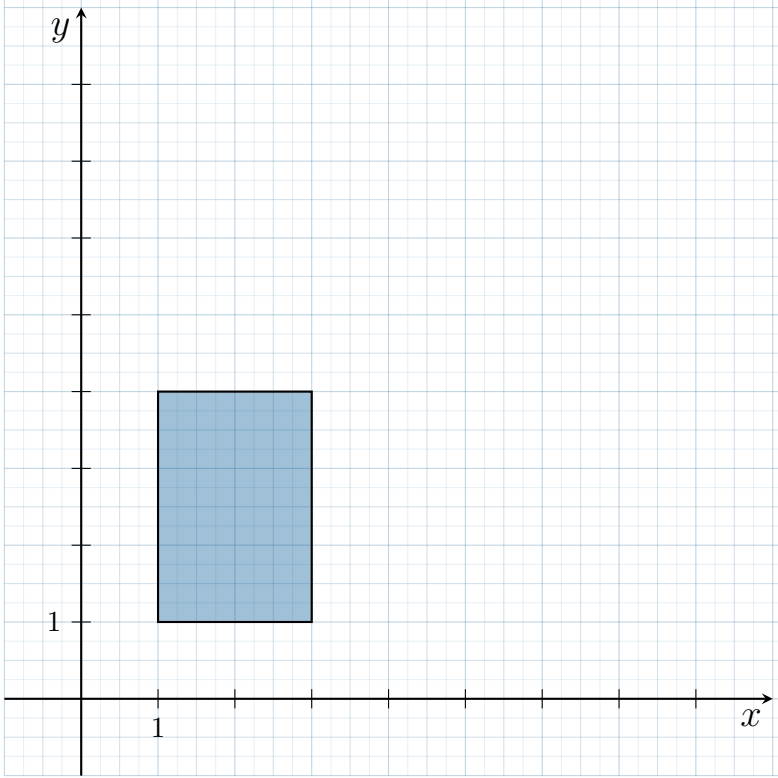
Sketch the vector $(4,3)-(1,2)$ as an arrow and compare this arrow with an arrow that represents $(8,8)-(5,7)$. Write both differences in the vector notation, that is as $\langle a,b\rangle$ for appropriate choices of a and b.



Exercise 4

(a) Show graphically how $\langle 5, 2 \rangle$ moves the rectangle R sketched below.

(b) Use set builder notation to describe the set R as well as the set $\langle 5, 2 \rangle + R$ and show how the addition of the vector to R moves R.



Exercise 5

Take f to be the function that is given by the formula

$$f(x) = x^2 - x + 1$$

with domain $(-1, 3]$.

(a) Use set builder notation to describe f as a subset of $\mathbb{R}^2$.

(b) Take g to be the subset of $\mathbb{R}^2$ that is given by $\langle 2, -1 \rangle + f$. Write in plain English what this sum means.

(c) The set g is a function. Identify the domain of g and a formula for $g(x)$ for each x in the domain of g.

Exercise 6

Take S to be the solution set to the equation

$$y^2 - x^3 - 2x^2 = 0.$$

(a) Identify an equation for the set $\langle -2, 3 \rangle + S$.

(b) Use a graphing application to sketch S and $\langle -2, 3 \rangle + S$.

(c) For any real numbers a and b, identify a formula for the set $\langle a, b \rangle + S$ and use a graphing application to sketch this set.