Linguistic Mapping

The Principles of Calculus I

I

Decomposition

I.6

Piecewise Functions

Classroom Exercises

Copyright © 2024 by Bryan Carrillo and David Weisbart.

All rights reserved. No part of this publication may be reproduced or transmitted in any form or by any means, electronic or mechanical, including photocopy, recording, or any information storage and retrieval system, without permission in writing from Bryan Carrillo and David Weisbart.

Exercise 1

Take f to be the function that is given by

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x < -4 \\ 3x - 1 & \text{if } 3 \leq x < 8 \\ 5x^2 & \text{if } x \geq 8. \end{cases}$$

For any subset D of $\mathbb{R}$, explain in plain English the meaning of the symbol $f\big|_D$. How do you read this symbol?

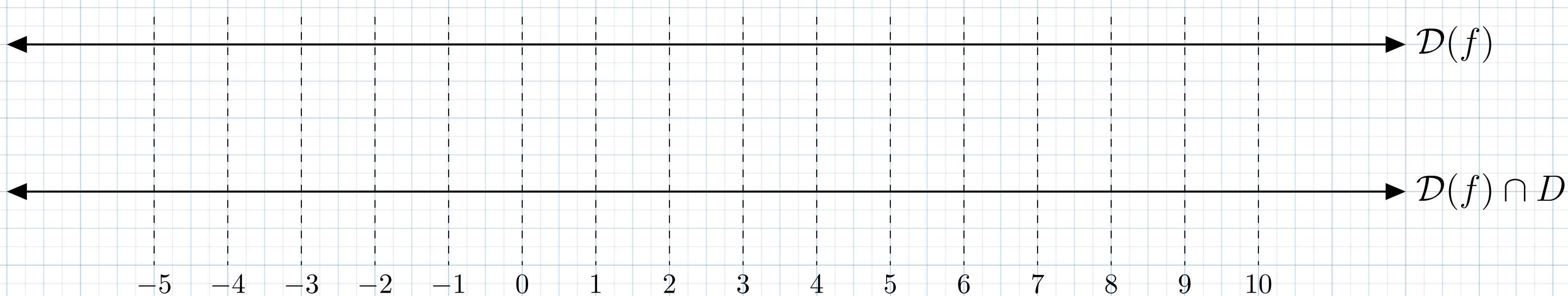
Copyright © 2024 by Bryan Carrillo and David Weisbart.

Take f to be the function that is given by

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x < -4 \\ 3x - 1 & \text{if } 3 \leq x < 8 \\ 5x^2 & \text{if } x \geq 8. \end{cases}$$

For each of these choices of interval D, identify the domain of the function $f\big|_D$ as well as a formula for this function:

(a) $D = (-\infty, -4)$;



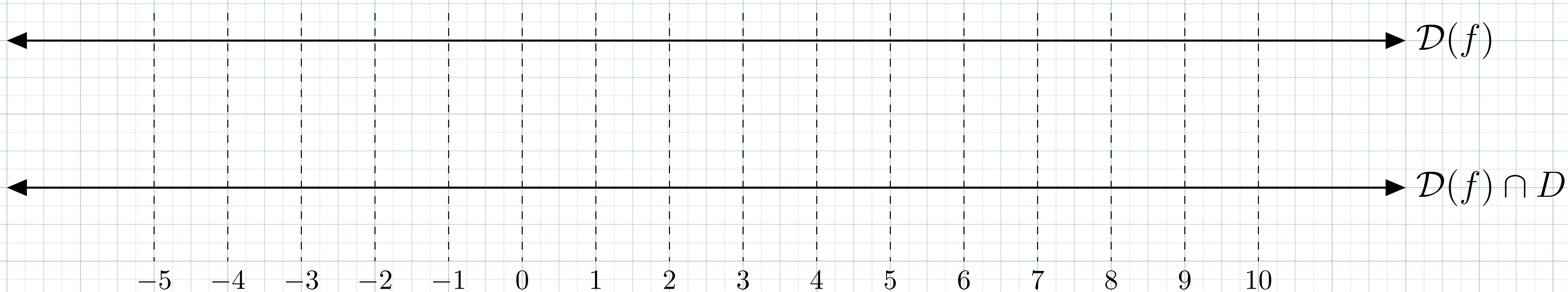
Copyright © 2024 by Bryan Carrillo and David Weisbart.

Take f to be the function that is given by

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x < -4 \\ 3x - 1 & \text{if } 3 \leq x < 8 \\ 5x^2 & \text{if } x \geq 8. \end{cases}$$

For each of these choices of interval D, identify the domain of the function $f\big|_D$ as well as a formula for this function:

(b) $D = [3, 8)$;



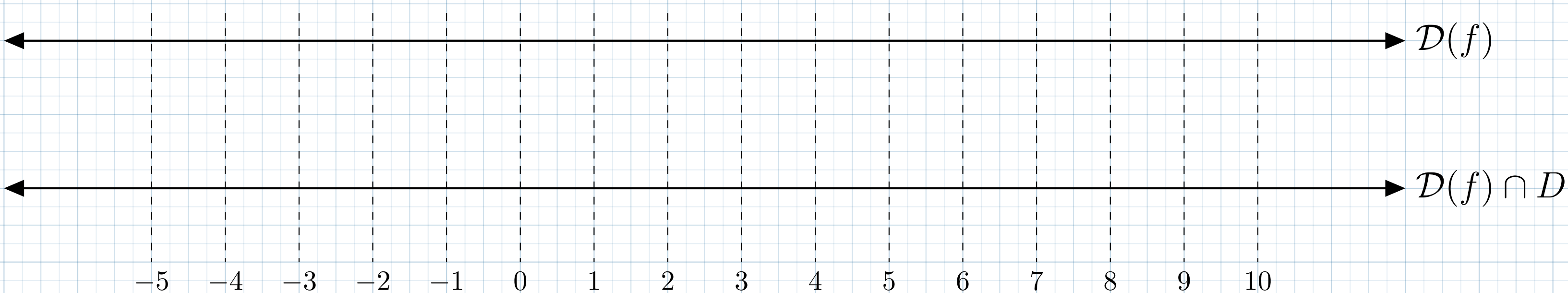
Copyright © 2024 by Bryan Carrillo and David Weisbart.

Take f to be the function that is given by

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x < -4 \\ 3x - 1 & \text{if } 3 \leq x < 8 \\ 5x^2 & \text{if } x \geq 8. \end{cases}$$

For each of these choices of interval D, identify the domain of the function $f\big|_D$ as well as a formula for this function:

(c) $D = [8, \infty)$;



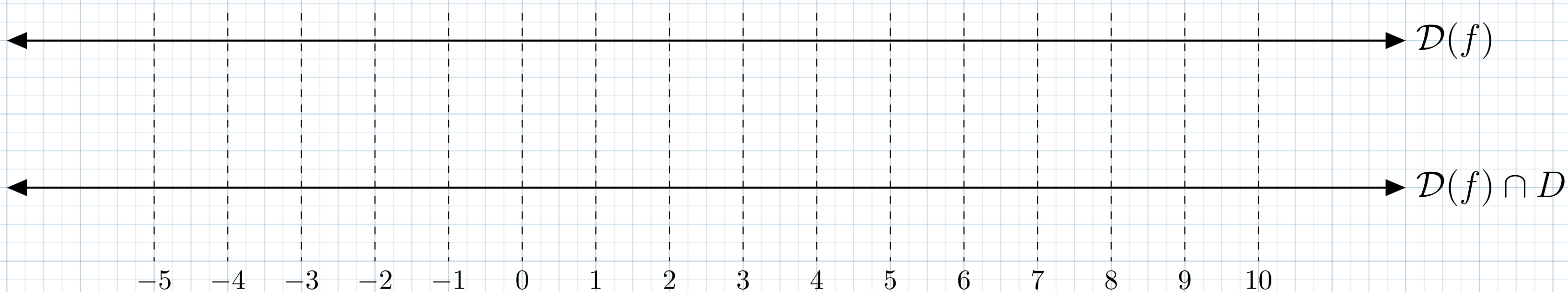
Copyright © 2024 by Bryan Carrillo and David Weisbart.

Take f to be the function that is given by

$$f(x) = \begin{cases} x^2+1 & \text{if } x < -4 \\ 3x-1 & \text{if } 3 \le x < 8 \\ 5x^2 & \text{if } x \ge 8. \end{cases}$$

For each of these choices of interval D, identify the domain of the function $f\big|_D$ as well as a formula for this function:

(d) $D = [6, 10)$.



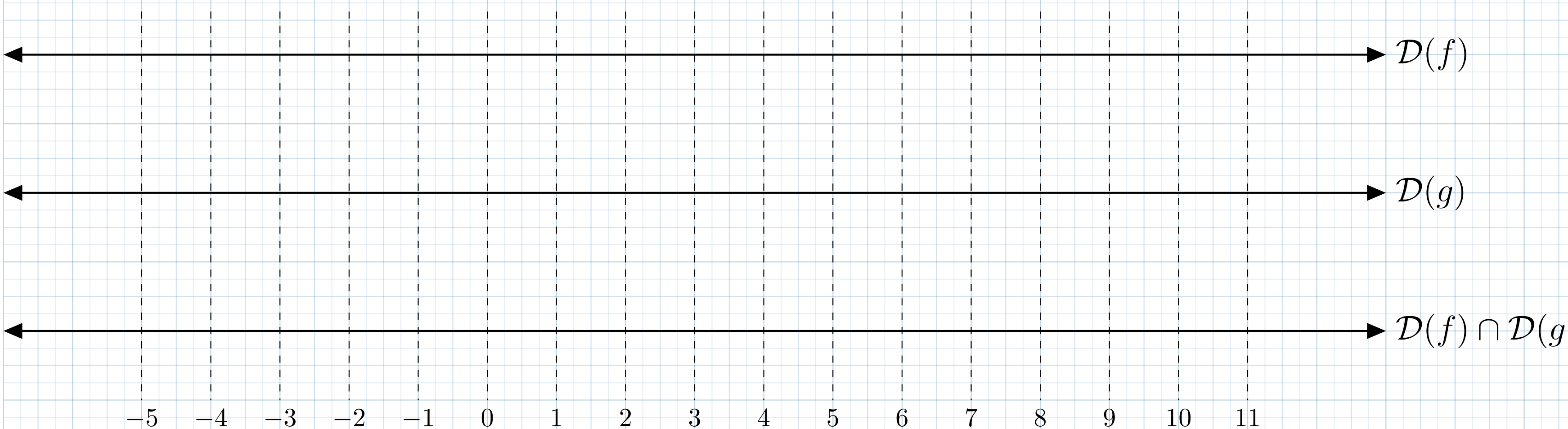
Copyright © 2024 by Bryan Carrillo and David Weisbart.

Exercise 2

Take f and g to be the functions that are given by

$$f(x) = \begin{cases} 2x+1 & \text{if } x < -3 \\ x & \text{if } -1 < x < 4 \\ x-5 & \text{if } x > 7 \end{cases} \quad \text{and} \quad g(x) = \begin{cases} 2 & \text{if } -5 < x \leq 0 \\ x^2+1 & \text{if } 0 < x \leq 5 \\ -x+8 & \text{if } 6 \leq x \leq 10 \\ x+1 & \text{if } x > 10. \end{cases}$$

Sketch the domains of f and g as subsets of the real line and determine their intersection.



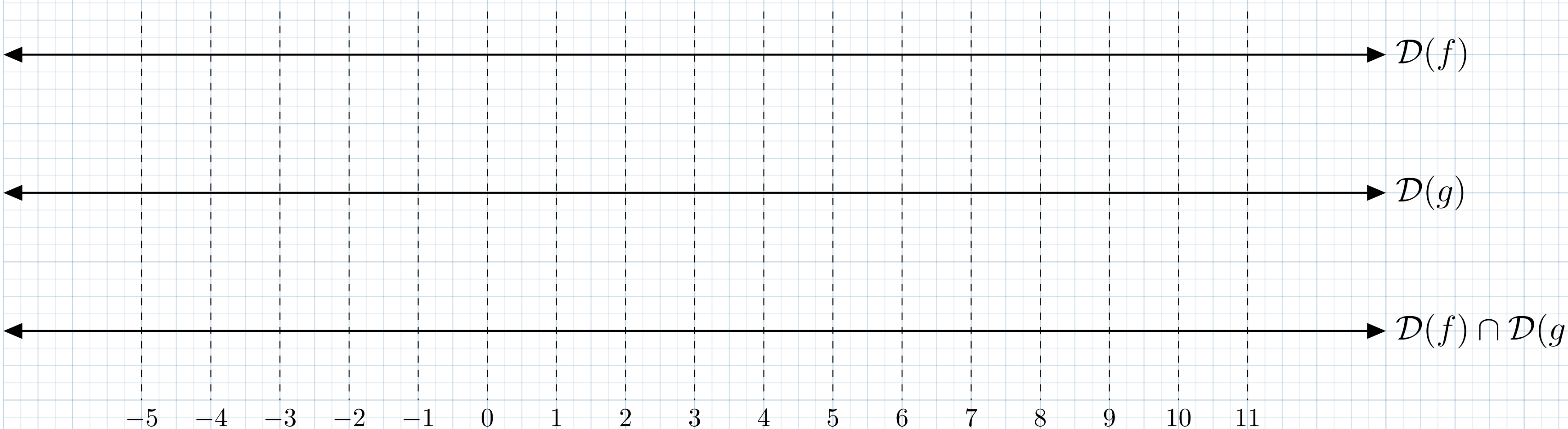
Copyright © 2024 by Bryan Carrillo and David Weisbart.

Exercise 3

Take f and g to be the functions that are given by

$$f(x) = \begin{cases} 2x+1 & \text{if } x < -3 \\ x & \text{if } -1 < x < 4 \\ x-5 & \text{if } x > 7 \end{cases} \quad \text{and} \quad g(x) = \begin{cases} 2 & \text{if } -5 < x \le 0 \\ x^2+1 & \text{if } 0 < x \le 5 \\ -x+8 & \text{if } 6 \le x \le 10 \\ x+1 & \text{if } x > 10. \end{cases}$$

Determine a commensurable partition for the restrictions of f and g to $\mathcal{D}(f) \cap \mathcal{D}(g)$.



Copyright © 2024 by Bryan Carrillo and David Weisbart.

Copyright © 2024 by Bryan Carrillo and David Weisbart.

Exercise 4

Take f and g to be the functions that are given by

$$f(x) = \begin{cases} 2x+1 & \text{if } x < -3 \\ x & \text{if } -1 < x < 4 \\ x-5 & \text{if } x > 7 \end{cases} \quad \text{and} \quad g(x) = \begin{cases} 2 & \text{if } -5 < x \leq 0 \\ x^2+1 & \text{if } 0 < x \leq 5 \\ -x+8 & \text{if } 6 \leq x \leq 10 \\ x+1 & \text{if } x > 10. \end{cases}$$

Express both $f+g$ and $\frac{f}{g}$ as piecewise functions and explain how you used the idea of a commensurable partition to determine these functions.

Copyright © 2024 by Bryan Carrillo and David Weisbart.

Copyright © 2024 by Bryan Carrillo and David Weisbart.

Exercise 5

Take f and g to be the functions that are given by

$$f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ 3x+1 & \text{if } 4 \leq x < 8 \end{cases} \quad \text{and} \quad g(x) = \begin{cases} x-1 & \text{if } -2 \leq x < 6 \\ -x+18 & \text{if } 9 \leq x \leq 27. \end{cases}$$

Describe $f(\square)$ piecewise with respect to the argument $\square$, and then describe $f(g(x))$ piecewise with respect to the argument $g(x)$.

Copyright © 2024 by Bryan Carrillo and David Weisbart.

Exercise 6

Take f and g to be the functions that are given by

$$f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ 3x+1 & \text{if } 4 \leq x < 8 \end{cases} \quad \text{and} \quad g(x) = \begin{cases} x-1 & \text{if } -2 \leq x < 6 \\ -x+18 & \text{if } 9 \leq x \leq 27. \end{cases}$$

Identify all solutions to the inequality

$$g(x) < 1.$$

Copyright © 2024 by Bryan Carrillo and David Weisbart.

Exercise 7

Take f and g to be the functions that are given by

$$f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ 3x+1 & \text{if } 4 \leq x < 8 \end{cases} \quad \text{and} \quad g(x) = \begin{cases} x-1 & \text{if } -2 \leq x < 6 \\ -x+18 & \text{if } 9 \leq x \leq 27. \end{cases}$$

Identify all solutions to the inequality

$$g(x) \geq 4.$$

Copyright © 2024 by Bryan Carrillo and David Weisbart.

Exercise 8

Take f and g to be the functions that are given by

$$f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ 3x+1 & \text{if } 4 \le x < 8 \end{cases} \quad \text{and} \quad g(x) = \begin{cases} x-1 & \text{if } -2 \le x < 6 \\ -x+18 & \text{if } 9 \le x \le 27. \end{cases}$$

Identify all solutions to the inequality

$$g(x) < 8.$$

Copyright © 2024 by Bryan Carrillo and David Weisbart.

Exercise 9

Take f and g to be the functions that are given by

$$f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ 3x+1 & \text{if } 4 \leq x < 8 \end{cases} \quad \text{and} \quad g(x) = \begin{cases} x-1 & \text{if } -2 \leq x < 6 \\ -x+18 & \text{if } 9 \leq x \leq 27. \end{cases}$$

The solution to the compound inequality

$$4 \leq g(x) < 8$$

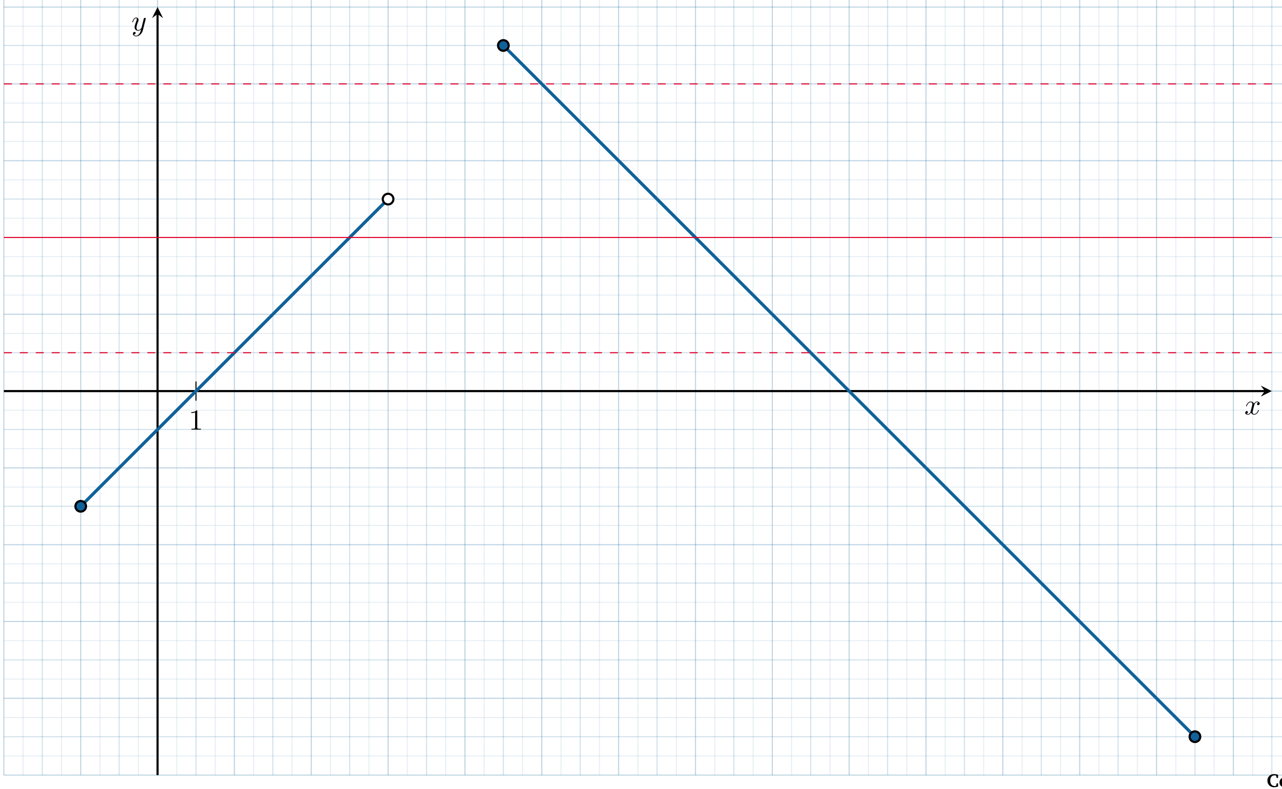
is an intersection of sets. Explain what this means and identify all solutions to the given compound inequality.

Copyright © 2024 by Bryan Carrillo and David Weisbart.

Copyright © 2024 by Bryan Carrillo and David Weisbart.

Use the graph immediately below to visualize the solution set to the inequalities

$$g(x) < 1 \quad \textbf{or} \quad 4 \leq g(x) < 8, \quad \textbf{where} \quad g(x) = \begin{cases} x - 1 & \textbf{if } -2 \leq x < 6 \\ -x + 18 & \textbf{if } 9 \leq x \leq 27. \end{cases}$$



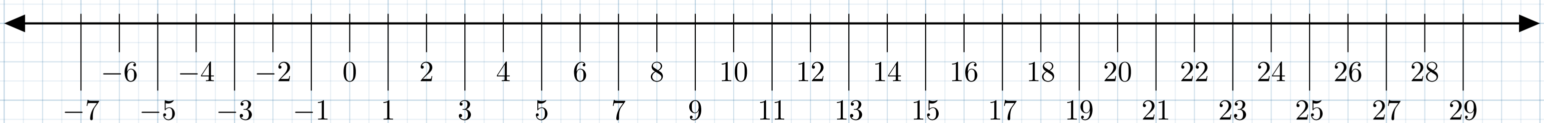
Copyright © 2024 by Bryan Carrillo and David Weisbart.

Exercise 11

Sketch the solution set to the inequalities

$$g(x) < 1 \quad \textbf{or} \quad 4 \le g(x) < 8, \quad \textbf{where} \quad g(x) = \begin{cases} x - 1 & \text{if } -2 \le x < 6 \\ -x + 18 & \text{if } 9 \le x \le 27. \end{cases}$$

Why was it useful to visualize the solution set as a subset of the plane, but important for us to distinguish between the useful representation and the second sketch as a subset of the real line?



Copyright © 2024 by Bryan Carrillo and David Weisbart.

Exercise 12

Take f and g to be the functions that are given by

$$f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ 3x+1 & \text{if } 4 \le x < 8 \end{cases} \quad \text{and} \quad g(x) = \begin{cases} x-1 & \text{if } -2 \le x < 6 \\ -x+18 & \text{if } 9 \le x \le 27. \end{cases}$$

Write $f \circ g$ as a piecewise function and determine the domain of $f \circ g$.

Copyright © 2024 by Bryan Carrillo and David Weisbart.

Copyright © 2024 by Bryan Carrillo and David Weisbart.