Linguistic Mapping

# The Principles of Calculus I

I

Decomposition

I.5

Manipulating Functions
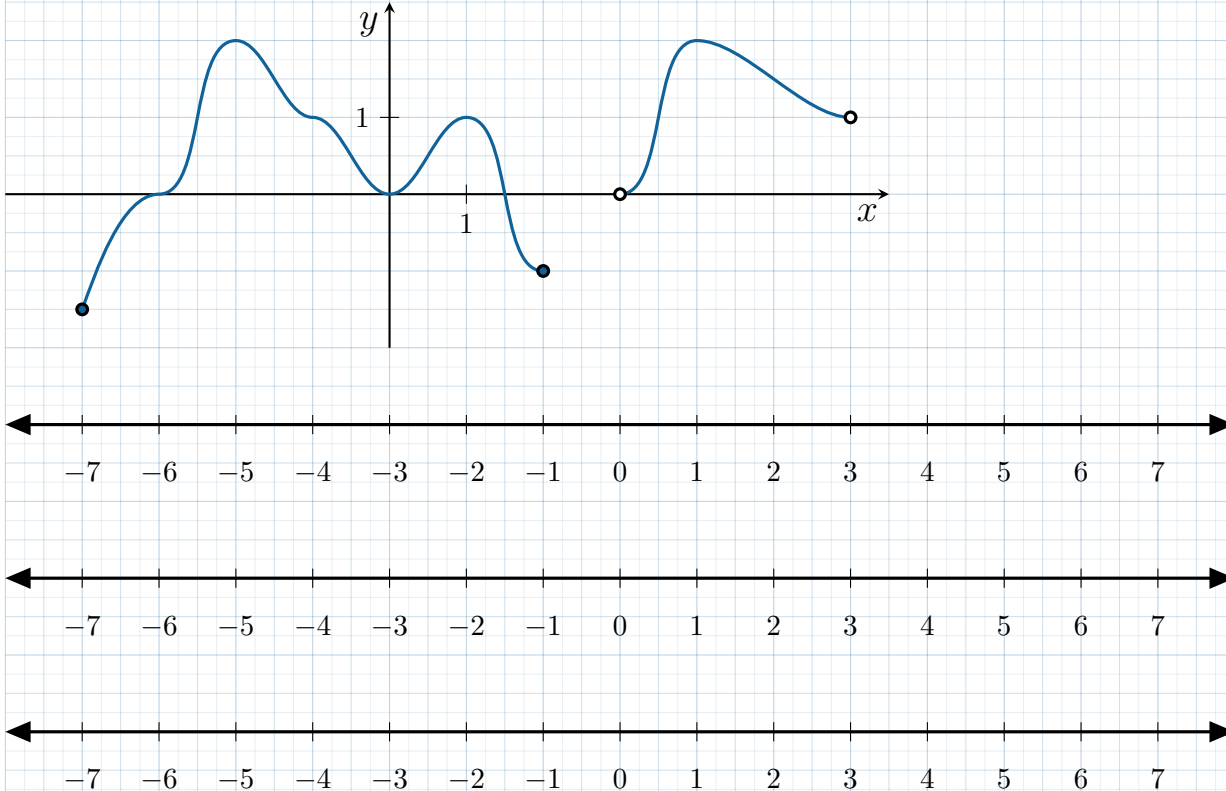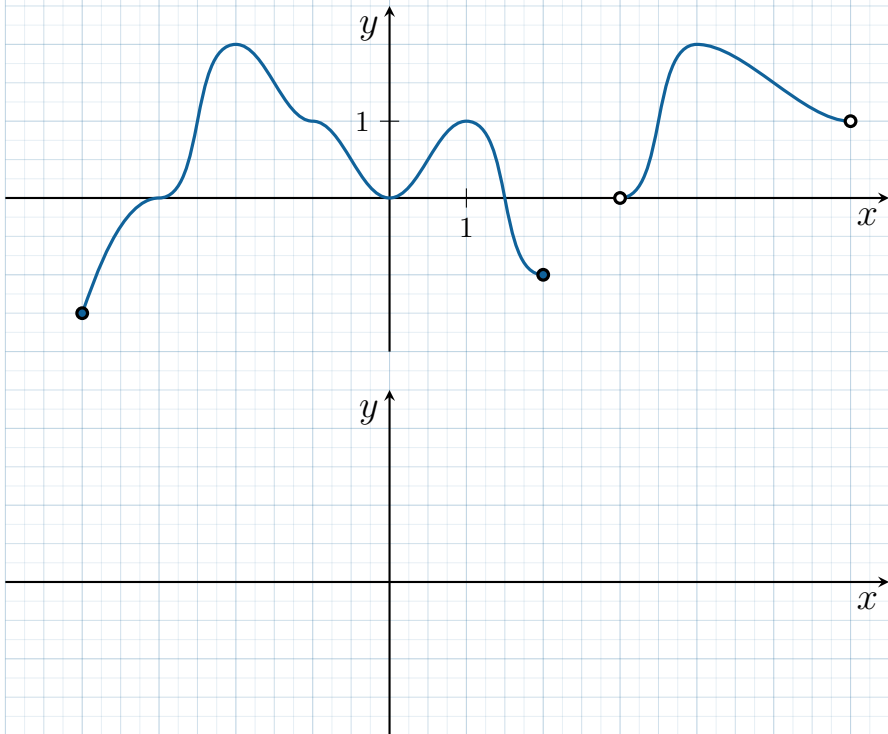
Classroom Exercises

# Exercise 1

Sketch on a real number line the domain of the function $f$ that is given below and sketch the intersection of this domain with the set $(-3, 1] \cup [4, 5)$.

Sketch the restriction of the function $f$ to the set $(-3, 1] \cup [4, 5)$, where $f$ is given below.

How do you read the notation $g = f\big|_{(-3,1]\cup[4,5)}$?

# Exercise 3

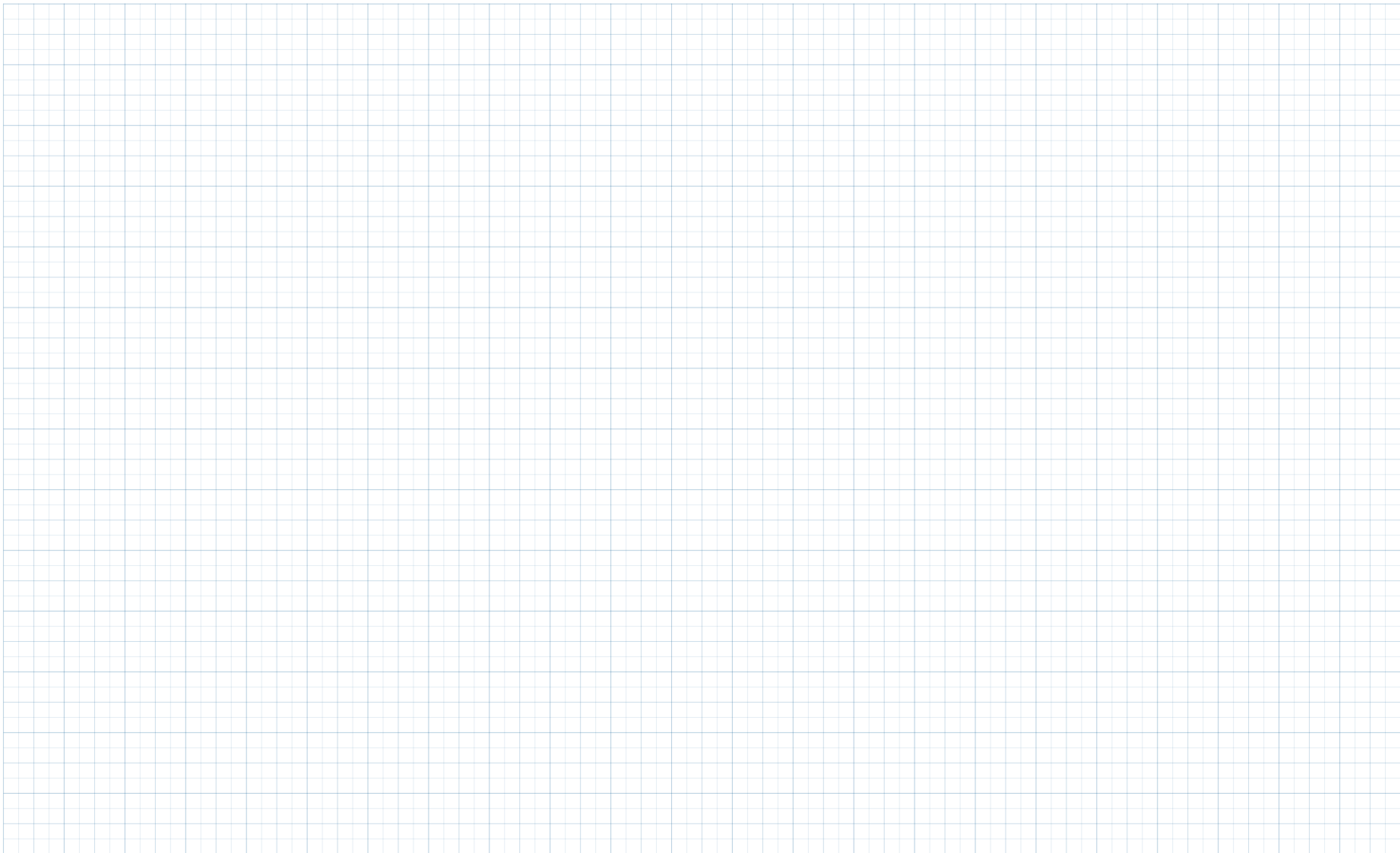Take $a$, $b$, $c$, $d$, and $e$ to be the functions that are given by

$$a(x) = x^2, \quad b(x) = 2x, \quad c(x) = x - 1, \quad d(x) = 5, \quad \textbf{and} \quad e(x) = 3x + 1.$$

Write in plain English and in the most elementary way possible a set of instructions for how to calculate these quantities:

**(a)** $5a(1) + b(1)$;

**(b)** $c(2)d(2) + 2$;

**(c)** $\frac{d(3)+4}{e(3)}$.

Take $a$, $b$, $c$, $d$, and $e$ to be the functions that are given by

$$a(x) = x^2, \quad b(x) = 2x, \quad c(x) = x - 1, \quad d(x) = 5, \quad \text{and} \quad e(x) = 3x + 1.$$

Calculate the quantity

$$\frac{5a(2) + b(2)}{c(2)} + d(2)e(2).$$

Take $a$, $b$, $c$, $d$, and $e$ to be the functions that are given by

$$a(x) = x^2, \quad b(x) = 2x, \quad c(x) = x - 1, \quad d(x) = 5, \quad \text{and} \quad e(x) = 3x + 1.$$

Identify a formula for the function $f$ that is given by

$$f(x) = \frac{5a(x) + b(x)}{c(x)} + d(x)e(x).$$

Take $a$ and $b$ to be the functions that are given by

$$a(x) = x^2 \quad \text{and} \quad b(x) = 2x - 3.$$

Determine the following values:

(a) $a(2)$;

(b) $a(\square)$ given that $\square$ is a real number;

(c) $a(b(1))$;

(d) $b(a(1))$;

(e) $a(b(x))$;

(f) $b(a(x))$.

## Exercise 7

The symbols $+$, $\cdot$, $\div$, and $\circ$ represent operations on pairs of functions that are referred to as summation, multiplication, division, and composition. Take $a$ and $b$ to be functions. For each of the parts below, write down the symbol as you would say it in plain English:

(a) $a + b$ [_____] ;

(b) $a \cdot b$ [_____] ;

(c) $\frac{a}{b}$ [_____] ;

(d) $\frac{b}{a}$ [_____] ;

(e) $a \circ b$ [_____] ;

(f) $b \circ a$ [_____] .

# Exercise 8

Take $a$ and $b$ to be functions and fill in the following blanks in the statements below:

(a) $a + b$ is the [                    ] of the functions [    ] and [    ] ;

(b) $a \cdot b$ is the [                    ] of the functions [    ] and [    ] ;

(c) $\frac{a}{b}$ is the [                ] of the function [    ] by [    ] ;

(d) $\frac{b}{a}$ is the [                ] of the function [    ] by [    ] ;

(e) $a \circ b$ is the [                ] of the function [    ] with [    ] ;

(f) $b \circ a$ is the [                ] of the function [    ] with [    ] .

# Exercise 9

Take $a$ and $b$ to be the functions that are given by

$$a(x) = x^2 \quad \text{and} \quad b(x) = 2x - 3.$$

Identify a formula for these functions:

**(a)** $a + b$;  **(c)** $\frac{a}{b}$;  **(e)** $b \circ a$.

**(b)** $a \cdot b$;  **(d)** $a \circ b$;

**(a)** $(a + b)(x) =$ 

**(b)** $(a \cdot b)(x) =$ 

**(c)** $\left(\frac{a}{b}\right)(x) =$ 

**(d)** $(a \circ b)(x) =$ 

**(e)** $(b \circ a)(x) =$

## Exercise 10

Decompose $f$ in terms of the functions $a$, $b$, $c$, $d$, and $e$ that are given by

$$a(x) = 3x, \quad b(x) = \sqrt{x}, \quad c(x) = x + 1, \quad d(x) = x^2, \quad \text{and} \quad e(x) = x + 5$$

for each of these choices of function $f$:

(a) $f(x) = x^2 + x + 5$;

(b) $f(x) = 3x\sqrt{x}$;

(c) $f(x) = \dfrac{x+1}{x^2}$;

(d) $f(x) = x^2 + 1$.

Take $f$ to be the function that is given by

$$f(x) = 3x\sqrt{\sqrt{(x+5)^2 + 1} + x^2} + \frac{3x(x+1)}{x^2 + 1}.$$

Decompose $f$ in terms of the functions $a$, $b$, $c$, $d$, and $e$ that are given by

$$a(x) = 3x, \quad b(x) = \sqrt{x}, \quad c(x) = x + 1, \quad d(x) = x^2, \quad \text{and} \quad e(x) = x + 5.$$