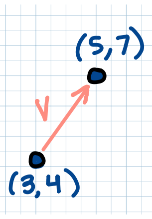
1. Identify a vector that moves $(3,4)$ to $(5,7)$.

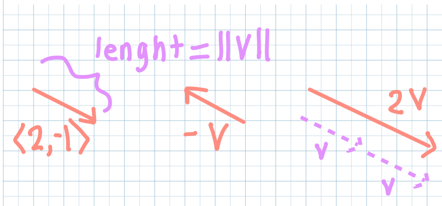


$V = (5,7) - (3,4) = \langle 2, 3 \rangle.$

2. Take V to be the vector given by

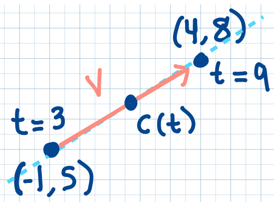
$$V = \langle 2, -1 \rangle.$$

Determine $2V$, $-V$, $\|V\|$, and $\hat{V}$ and sketch a picture to illustrate these quantities.



$2V = \langle 4,-2 \rangle$, $-V = \langle -2, 1 \rangle$, $\|V\| = \sqrt{2^2 + (-1)^2} = \sqrt{5}$, $\hat{V} = \frac{1}{\|V\|} V = \left\langle \frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}} \right\rangle$

3. A particle moves at constant velocity. It is at the point $(-1,5)$ at time 3 and at the point $(4,8)$ at time 9. Write an equation that models the motion of the particle.



$V = (4,8) - (-1,5) = \langle 5, 3 \rangle$

$c(t) = \frac{t-3}{6} \langle 5, 3 \rangle + (-1,5)$ which is equivalent to $c(t) = \left(\frac{5}{6}(t-3) - 1, \frac{1}{2}(t-3) + 5 \right)$

note: either equation is fine.

4. Write the following functions using set notation:

$$g(x) = x^3 + x, \quad h(x) = \log_2(x-2), \quad k(x) = 2^x - 1.$$

$g = \{ (x, x^3 + x) : x \in \mathbb{R} \}$ domain of g is $\mathbb{R}$

$h = \{ (x, \log_2(x-2)) : x > 2 \}$ domain of h is $x > 2$

$K = \{ (x, 2^x - 1) : x \in \mathbb{R} \}$ domain of K is $\mathbb{R}$

5. Sketch the function f that is given by

$$f(x) = \left(\frac{1}{2}\right)^{x+2} + 1.$$

Identify any asymptotes of the function.

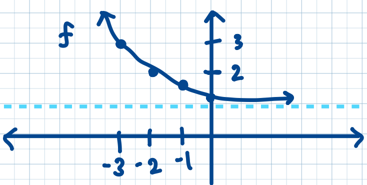
The function f decomposes like this:

$$f = \langle -2, 1\rangle + \exp_{\frac{1}{2}}, \quad f(x) = \left(\tfrac{1}{2}\right)^{\overbrace{x+2}^{\text{left } 2}} \underbrace{+1}_{\text{up } 1}$$

$$= \langle -2, 1\rangle + \{(x, (\tfrac{1}{2})^x) : x \in \mathbb{R}\}$$

$$= \{(x-2, (\tfrac{1}{2})^x + 1) : x \in \mathbb{R}\} \quad \text{set } \mathbb{X} = x-2 \text{ so } \mathbb{X}+2 = x$$

$$= \{(\mathbb{X}, \underbrace{(\tfrac{1}{2})^{\mathbb{X}+2} + 1}_{f(x)}) : \mathbb{X} \in \mathbb{R}\}$$



asymptote at $y=1$ Note: $\exp_{\frac{1}{2}}$ has HA at $y=0$, so HA of f is $\exp_{\frac{1}{2}}$'s HA shifted up 1

6. Sketch the function f that is given by

$$f(x) = \log_2(x-1) - 2.$$

Identify any asymptotes of the function.

The function f decomposes like this

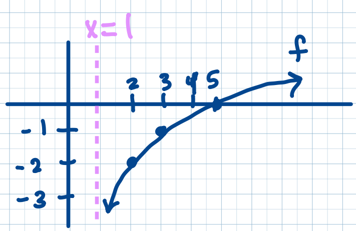
$$f = \langle 1, -2\rangle + \log_2, \quad f(x) = \log_2\overbrace{(x-1)}^{\text{right } 1}\underbrace{-2}_{\text{down } 2}$$

$$= \langle 1, -2\rangle + \{(x, \log_2(x)) : x > 0\}$$

$$= \{(x+1, \log_2(x) - 2) : x > 0\} \quad \text{Set } \mathbb{X} = x+1 \text{ or } \mathbb{X} - 1 = x$$

$$= \{(\mathbb{X}, \log_2(\mathbb{X}-1) - 2) : \mathbb{X} - 1 > 0\}$$

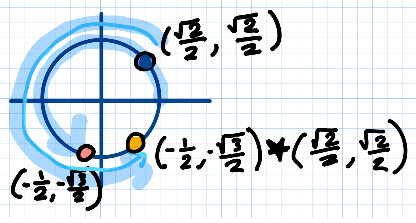
$$= \{(\mathbb{X}, \underbrace{\log_2(\mathbb{X}-1) - 2}_{f(x)}) : \mathbb{X} > 1\}$$



asymptote $x=1$ Note: $\log_2$ has VA $x=0$, so VA of f is $\log_2$'s VA shift right 1.

7. Calculate $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right) \star \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ and explain its meaning.

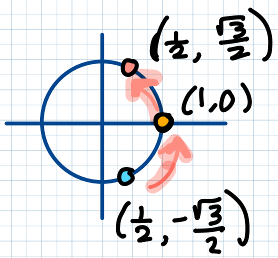
$$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right) \star \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) = \left(-\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}, -\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}\right) = \left(\frac{\sqrt{6}-\sqrt{2}}{4}, -\frac{\sqrt{2}+\sqrt{6}}{4}\right)$$



The meaning is rotating $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ by $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ or adding angles $\underset{45^\circ}{\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)}$ and $\underset{240^\circ}{\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)}$.

The result is $\left(\frac{\sqrt{6}-\sqrt{2}}{4}, -\frac{\sqrt{2}+\sqrt{6}}{4}\right)$ (corresponds to 285°)

8. Take p to be the point given by $p = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. Calculate p^{-1} and explain its meaning.



$$p^{-1} = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$$

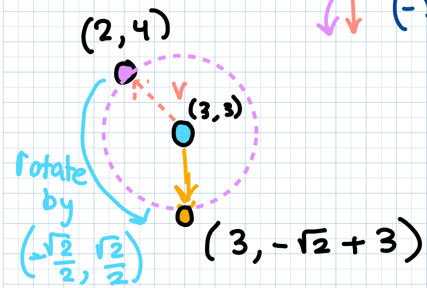
p^{-1} is the point on the unit circle so that

$$p^{-1} \star p = \underbrace{(1,0)}_{\text{identity}}$$

Rotating $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ by $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ produces $(1,0)$, the identity element.

9. Rotate the point $(2,4)$ about $(3,3)$ by $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

$$v = (2,4) - (3,3) = \langle -1, 1\rangle$$



$$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \star \langle -1, 1\rangle = \left\langle \left(-\frac{\sqrt{2}}{2}\right)\cdot(-1) - \frac{\sqrt{2}}{2}\cdot 1, \left(-\frac{\sqrt{2}}{2}\right)\cdot 1 + \left(\frac{\sqrt{2}}{2}\right)\cdot(-1)\right\rangle$$

$$= \left\langle \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right\rangle$$

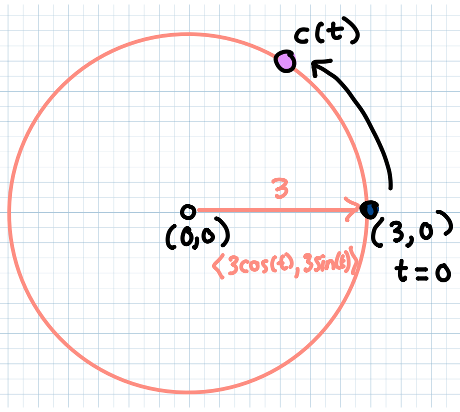
$$= \left\langle 0, -\frac{2\sqrt{2}}{2}\right\rangle$$

$$= \langle 0, -\sqrt{2}\rangle$$

$$\langle 0, -\sqrt{2}\rangle + (3,3) = (3, -\sqrt{2}+3)$$

Final Answer: $(3, -\sqrt{2}+3)$

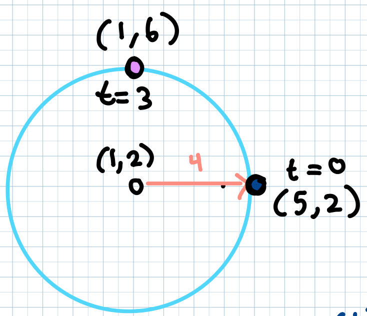
10. A particle rotates counterclockwise around the point $(0, 0)$ at constant speed. It is at the point $(3, 0)$ at time 0. Write an equation that models the position of the particle at time t.



$$c(t) = \langle 3\cos(t), 3\sin(t)\rangle + (0,0)$$

$$= (3\cos(t), 3\sin(t))$$

11. A particle rotates counterclockwise around the point $(1, 2)$ at constant speed. It is at the point $(5, 2)$ at time 0 and at the point $(1, 6)$ at time 3. Write an equation that models the position of the particle at time t.



$$c(t) = \langle 4\cos(d(t)), 4\sin(d(t))\rangle + (1,2)$$

where $d(0) = 0$ and $d(3) = \frac{\pi}{2}$

Take $d(t) = \frac{\pi}{6}t$ so

$$c(t) = \langle 4\cos(\tfrac{\pi}{6}t), 4\sin(\tfrac{\pi}{6}t)\rangle + (1,2) \text{ or } c(t) = \left(4\cos(\tfrac{\pi}{6}t) + 1, 4\sin(\tfrac{\pi}{6}t) + 2\right)$$

12. Take A and B to be two real numbers so that

$$\cos(A) = -\frac{1}{3}, \quad \sin(A) = \frac{2\sqrt{2}}{3}, \quad \cos(B) = \frac{2}{5} \quad \text{and} \quad \sin(B) = -\frac{\sqrt{21}}{5}.$$

Determine $\cos(A + B)$ and $\sin(A + B)$.

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

$$= -\frac{1}{3} \cdot \frac{2}{5} - \frac{2\sqrt{2}}{3} \cdot \left(-\frac{\sqrt{21}}{5}\right)$$

$$= \frac{2\sqrt{42} - 2}{15}$$

$$\sin(A+B) = \cos(A)\sin(B) + \sin(A)\cos(B)$$

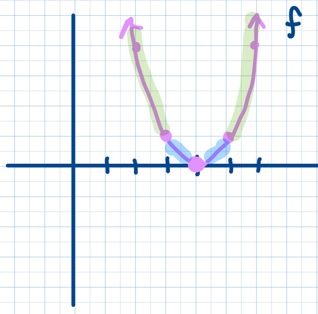
$$= -\frac{1}{3} \cdot \left(-\frac{\sqrt{21}}{5}\right) + \frac{2\sqrt{2}}{3} \cdot \frac{2}{5}$$

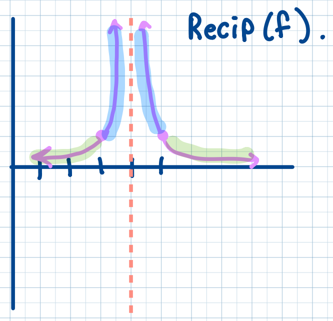
$$= \frac{\sqrt{21} + 4\sqrt{2}}{15}$$

13. Take f to be the function given by

$$f(x) = (x-4)^2.$$

Sketch $\mathrm{Recip}(f)$.





14. Take f to be the function whose sketch is given below. Sketch $\mathrm{Recip}(f)$.