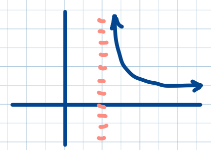
1. For each function f given below, compute the derivative of f to determine where f is increasing and decreasing:

a) $f(x) = \ln(x-1)$ on $I = (1, \infty)$

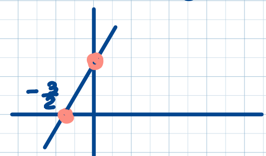
$f'(x) = \frac{1}{x-1}$



On I, $f' > 0$ so f is increasing on $(1, \infty)$.

b) $f(x) = x^2 + 3x$

$f'(x) = 2x + 3$



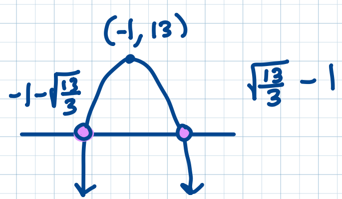
$f' > 0$ on $(-\frac{3}{2}, \infty)$ so f is increasing on $[-\frac{3}{2}, \infty)$.

$f' < 0$ on $(-\infty, -\frac{3}{2})$ so f is decreasing on $(-\infty, -\frac{3}{2}]$.

c) $f(x) = -x(x+5)(x-2)$

$$f'(x) = -(x+5)(x-2) - x(x-2) - x(x+5)$$
$$= -x^2 - 3x + 10 - x^2 + 2x - x^2 - 5x$$
$$= -3x^2 - 6x + 10$$
$$= -3(x+1)^2 + 13$$

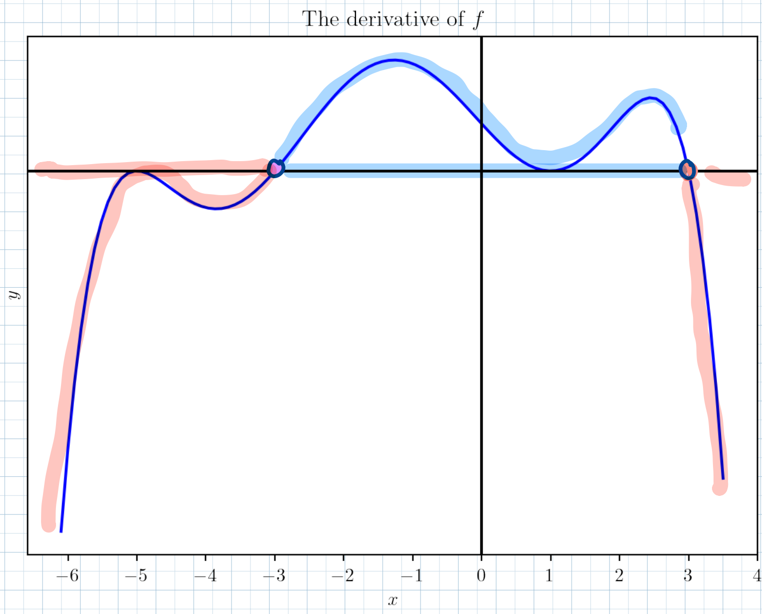
$$\begin{cases} -3 = A \\ -6 = -2Ah \\ 10 = Ah^2 + K \end{cases}$$

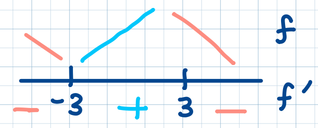


$f' > 0$ on $(-1-\sqrt{\frac{13}{3}}, \sqrt{\frac{13}{3}}+1)$ so f increasing on $[-1-\sqrt{\frac{13}{3}}, \sqrt{\frac{13}{3}}+1]$

$f' < 0$ on $(-\infty, -1-\sqrt{\frac{13}{3}}) \cup (\sqrt{\frac{13}{3}}-1, \infty)$ so f decreasing on $(-\infty, -1-\sqrt{\frac{13}{3}}]$, $[\sqrt{\frac{13}{3}}+1, \infty)$

2. Take f to be a differentiable function on $[-6, 3.5]$. Given this sketch of f' below, find and classify all extremal points of f in $(-6, 3.5)$. And find where f is increasing and decreasing in $(-6, 3.5)$.



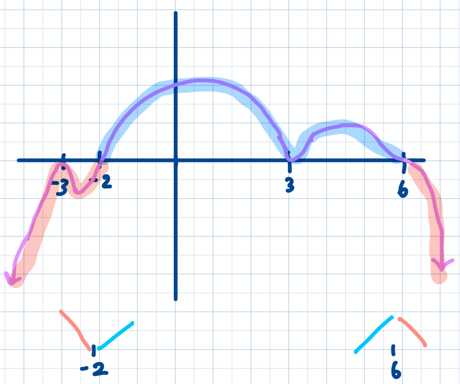


f has a local minimum at -3 and a local maximum at 3

f is increasing on $[-3, 3]$ and decreasing on $[-6, -3]$ and $[3, 3.5]$.

3. Take f to be a function f with $f'(x) = -(x+3)^2(x+2)(x-3)^4(x-6)^3$. Determine all points in $\mathbb{R}$ where f has a local maximum or minimum.

Sketch of f': $f' \sim -x^{10}$



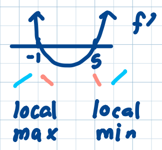
f has a local maximum at 6 and local minimum at -2.

4. Find and classify all extremal points for $f(x) = \frac{1}{3}x^3 - 2x^2 - 5x$ on $[-2, 6]$.

Because f is continuous on $[-2,6]$, f has a maximum and minimum in $[-2,6]$.

Identify critical points of f:

$f'(x) = x^2 - 4x - 5 = (x-5)(x+1)$ so $f' = 0$ if $x = 5$ or $x = -1$.



The function f attains its maximum or minimum on $\{-2, -1, 5, 6\}$.

$f(-2) = -\frac{2}{3}$

$f(-1) = \frac{8}{3}$

$f(5) = -\frac{100}{3}$

$f(6) = -30$

So f has a maximum at -1 and minimum at 5.

Also f has a local maximum at -1 and local maximum at 5.

5. For each function f that is given below, determine $f''(x)$:

a) $f(x) = 4^x + x^5 + e^2$

$f'(x) = \ln(4)4^x + 5x^4$

$f''(x) = (\ln(4))^2 4^x + 20x^3$

b) $f(x) = \frac{1}{x-2}\ln(x)$

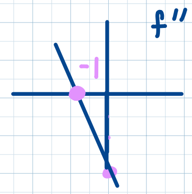
$f'(x) = -\frac{1}{(x-2)^2}\ln(x) + \frac{1}{x^2-2x}$

$f''(x) = \frac{2}{(x-2)^3}\ln(x) - \frac{1}{x(x-2)^2} - \frac{2x-2}{(x^2-2x)^2}$

6. Take $f(x) = -\frac{1}{3}x^3 - x^2 + 3x + 4$. Find all inflection points of f and give all maximal intervals on which f is convex or concave.

Analyze f'':

$f'(x) = -x^2 - 2x + 3$, $f''(x) = -2x - 2$

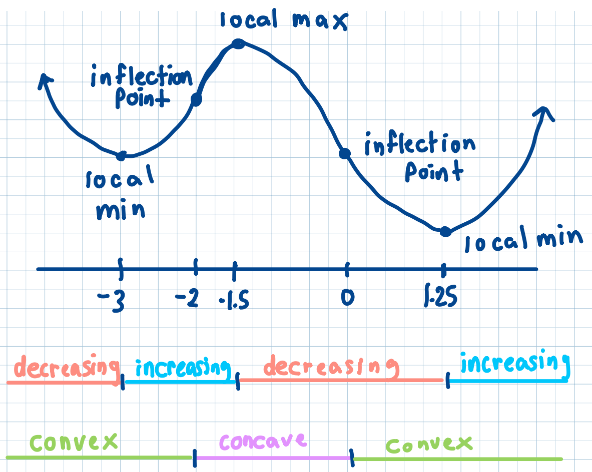


$f'' > 0$ on $(-\infty, -1)$ so f is convex (concave up) on $(-\infty, 1]$

$f'' < 0$ on $(-1, \infty)$ so f is concave (concave down) on $[-1, \infty)$

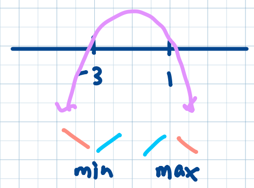
7. Sketch an example of a continuous function f that has the property that

- f'' is positive on $(-\infty, -2) \cup (0, \infty)$,
- f'' is negative on $(-2, 0)$,
- the zero set of f'' is $\{-2, 0\}$,
- f' is negative on $(-\infty, -3) \cup (-1.5, 1.25)$,
- f' is positive on $(-3, -1.5) \cup (1.25, \infty)$,
- the zero set of f' is $\{-3, -1.5, 1.25\}$.



8. Sketch the function $f(x) = -\frac{1}{3}x^3 - x^2 + 3x + 4$ by determining where f is increasing, decreasing, convex, concave, and where f has a local max, local min, and inflection points.

$f'(x) = -x^2 - 2x + 3 = -(x+3)(x-1)$



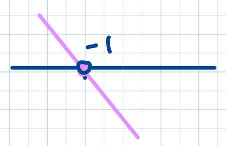
$f' > 0$ on $(-3, 1)$ so f increases on $[-3, 1]$.

$f' < 0$ on $(-\infty, -3) \cup (1, \infty)$ so f decreases on $(-\infty, -3]$ and $[1, \infty)$.

f has a local minimum at -3.

f has a local maximum at 1.

$f''(x) = -2x - 2$



$f'' > 0$ on $(-\infty, -1)$ so f is convex on $[-\infty, -1]$.

$f'' < 0$ on $(-1, \infty)$ so f is concave on $[-1, \infty)$.

f has an inflection point at -1.

Sketch of f: