1. Use Fermat's theorem to determine all points at which each function f below can potentially attain a local maximum or a local minimum:

a) $f(x) = -4x^2 + 4x + 3$

b) $f(x) = x^{\frac{2}{3}} + x$

2. Take f to be the function that is given by

$$f(x) = -4x^2 + 4x + 3.$$

Determine the maximum and minimum values that f attains on $[0, 2]$.

3. Take $f(x) = \frac{1}{4}x^2 + \frac{1}{4}$. This is a differentiable function on the interval $(-1, 2)$ and a continuous function on the interval $[-1, 2]$.

a) Find a value M so that $|f'(x)| \leq M$ for all x in $[-1, 2]$.

b) Given that $f(-1) = \frac{1}{2}$, find the smallest range of values that is guaranteed to contain $f([-1, 2])$ and sketch the smallest region that you can that is guaranteed to contain f. Then compare it to the graph of f.

c) Given that $f(-1) = \frac{1}{2}$ and $f(2) = \frac{5}{4}$, find the smallest range of values that is guaranteed to contain $f([-1, 2])$ and sketch the smallest region that you can that is guaranteed to contain f. Then compare it to the graph of f.

4. Take f and g to be differentiable functions on the interval $[1, 9]$ so that $f(2) = 4$, $g(2) = 8$, and that for any x in $(1, 9)$,

$$f'(x) - g'(x) = 0.$$

Determine $g(9) - f(9)$.

5. For each function f that is given below, determine the antiderivative of f :

a) $f(x) = 20x^3 + \mathrm{e}^x + 2^x + 4\cos(x)$

b) $f(x) = \frac{3}{x} + \frac{40}{x^2+1} - \ln(2)\sqrt{x} + 2$

6. Take f to be the function with the property that

$$\int f(x)\,\mathrm{d}x = \sin(x) + \tan(x) + 2x^3 + C.$$

Identify the function f.

7. Calculate the following:

a) $\displaystyle\int_1^2 x^5\,\mathrm{d}x$

b) $\displaystyle\int_0^1 (20x^3 + \mathrm{e}^x + 2^x + 4\cos(x))\,\mathrm{d}x$

c) Derivative of F where $F(x) = \displaystyle\int_2^x \sin(t^2)\,\mathrm{d}t$.

d) Derivative of F where $F(x) = \displaystyle\int_2^{4x+2} \sin(t^2)\,\mathrm{d}t$.

8. Determine the following:

a) $\displaystyle\int 4x^3 \sin(x^4)\,\mathrm{d}x$

b) $\displaystyle\int (40x^3 + x)\sqrt{20x^4 + x^2 + 1}\,\mathrm{d}x$

c) $\displaystyle\int \frac{\mathrm{e}^x + \ln(2)2^x}{\mathrm{e}^x + 2^x}\,\mathrm{d}x$

9. Use the fact that

$$(x\cos(x))' = \cos(x) - x\sin(x)$$

to determine

$$\int x\sin(x)\,\mathrm{d}x.$$

10. The function $v(t)$ given by $v(t) = \langle 5t + 2, 5\rangle$ is the velocity of a path c at time t. Given that $c(2) = (4, 5)$, reconstruct the path c and simulate the motion of the particle together with the particle's velocity vector.

11 Verify that the following are of indeterminate forms and use L'Hopital's rule to determine the given limits:

a) $\lim_{x \to 5} \frac{x^3 - 24x - 5}{x - 5}$

b) $\lim_{x \to \infty} \left(\sqrt{4x^2 + 3x + 1} - 2x \right)$

c) $\lim_{x \to \infty} \frac{x^3}{e^x}$