1. Take f to be the function that is given by

$$f(x) = 5x + 2 + E_{-1}(x)$$

where E_{-1} is $o(x+1)$. Determine $f'(-1)$ and determine an equation for the line L that is tangent to f at $(-1, f(-1))$.

Local linear approximation of f is $5x+2$. This is the line tangent to f at $(-1,3)$ and $f'(-1)=5$.

2. Take $f(x) = x^3$. Write the local linear approximation of f at $a = 1$. Use your local linear approximation to approximate $(1.05)^3$.

Local linear approximation at $a=1$ is $L(x) = f(1) + f'(1)(x-1)$:

$f(1) = 1$
$f'(x) = 3x^2 \Rightarrow L(x) = 1 + 3(x-1)$.
$f'(1) = 3$

Hence

$$(1.05)^3 = f(1.05) \approx L(1.05) = 1 + 3(1.05 - 1) = 1.15.$$

3. The following functions are continuous at $x_0 = 1$, but are not differentiable at $x_0 = 1$. Explain why.

a) $f(x) = |x - 1|$

Continuous at $x=1$ since

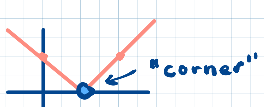
$$\lim_{x\to 1} |x-1| = 0 = f(1).$$

Not differentiable because

$$\lim_{h\to 0^-} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0^-} \frac{|h|}{h} = -1$$

$$\lim_{h\to 0^+} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0^+} \frac{|h|}{h} = 1$$

So $f'(1)$ does not exist



b) $f(x) = \sqrt{x-1}$

Continuous at $x=1$ since

$$\lim_{x\to 1^+} \sqrt{x-1} = 0 = f(1)$$

Not differentiable because

$$\lim_{h\to 0^+} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0^+} \frac{\sqrt{h}}{h} = \lim_{h\to 0^+} \frac{1}{\sqrt{h}} = \infty$$

So $f'(1)$ does not exist.

4. Calculate the derivative of each function by decomposing it into a sum and or product of simpler functions and by using the appropriate derivative rule.

a) $f(x) = 5x^4 + \sin(x) - \mathrm{e}$

$f = 5\,\mathrm{pow}_4 + \sin + e$

So

$f'(x) = 5\,\mathrm{pow}_4'(x) + \sin'(x) + (e)'$

$= 5 \cdot 4\,\mathrm{pow}_3(x) + \cos(x) + 0$

$= 20x^3 + \cos(x)$.

b) $f(x) = x^{4/5}\exp_5(x) + \sqrt{x}\tan(x)$

$f = \underbrace{\mathrm{pow}_{\frac{4}{5}} \cdot \exp_5}_{\text{product}} + \underbrace{\mathrm{pow}_{\frac{1}{2}} \tan(x)}_{\text{product}}$

So

$f'(x) = \mathrm{pow}_{\frac{4}{5}}'(x)\exp_5(x) + \mathrm{pow}_{\frac{4}{5}}\exp_5'(x)$

$+ \mathrm{pow}_{\frac{1}{2}}'(x)\tan(x) + \mathrm{pow}_{\frac{1}{2}}(x)\tan'(x)$

$= \frac{4}{5}x^{-\frac{1}{5}}5^x + x^{\frac{4}{5}}\ln(5)5^x + \frac{1}{2\sqrt{x}}\tan(x)$

$+ \sqrt{x}\sec^2(x)$.

5. Calculate the derivative of each function f.

a) $f(x) = \frac{1}{\sin(x)}$ $= \csc(x)$

$f = \text{Recipo} \sin$

So

$f'(x) = -\frac{(\sin(x))'}{\sin^2(x)}$ Reciprocal Rule

$= -\frac{\cos(x)}{\sin^2(x)}$

$= -\frac{\cos(x)}{\sin(x)} \frac{1}{\sin(x)}$

$= -\cot(x)\csc(x)$.

b) $f(x) = \frac{x+3}{\ln(x)}$

$f = \frac{\mathrm{pow}_1 + 3}{\ln}$ Quotient

So

$f'(x) = \frac{(x+3)'\ln(x) - (x+3)\ln'(x)}{(\ln(x))^2}$

$= \frac{\ln(x) - \frac{(x+3)}{x}}{(\ln(x))^2}$.

c) $f(x) = \exp(x) \cdot \frac{2x}{\exp(x)+\cos(x)}$

$f = \underbrace{\exp \circ \frac{2\,\mathrm{pow}_1}{\exp + \cos}}_{\text{Product}}$ Quotient

So

$f'(x) = (e^x)' \cdot \frac{2x}{e^x + \cos(x)} + e^x \cdot \left(\frac{2x}{e^x + \cos(x)}\right)'$ Product

$= (e^x)' \cdot \frac{2x}{e^x + \cos(x)} + e^x \cdot \frac{(2x)'(e^x + \cos(x)) - (2x)(e^x + \cos(x))'}{(e^x + \cos(x))^2}$ Quotient

$= \frac{2xe^x}{e^x + \cos(x)} + e^x \cdot \frac{2(e^x + \cos(x)) - 2x(e^x - \sin(x))}{(e^x + \cos(x))^2}$.

6. Calculate the derivative of each function f.

a) $f(x) = \sqrt{4\cos(x) + \ln(x)}$

$$f = \underbrace{pow_{\frac{1}{2}} \circ (4\cos + \ln)}_{\text{chain}}$$

So

$$f'(x) = pow'_{\frac{1}{2}}(4\cos(x) + \ln(x)) \cdot (4\cos(x) + \ln(x))' \quad \text{chain}$$

$$= \frac{1}{2} pow_{-\frac{1}{2}}(4\cos(x) + \ln(x)) \cdot \left(-4\sin(x) + \frac{1}{x}\right)$$

$$= \frac{-4\sin(x) + \frac{1}{x}}{2\sqrt{4\cos(x) + \ln(x)}}$$

b) $f(x) = \sin(e^x)\cos(3x + 1)$

$$f = \underbrace{(\underbrace{\sin \circ \exp}_{\text{chain}}) \bullet (\underbrace{\cos \circ (3pow_1 + 1)}_{\text{chain}})}_{\text{Product}}$$

So

$$f'(x) = (\sin(e^x))'(\cos(3x+1)) + \sin(e^x)(\cos(3x+1))'$$

$$= \cos(e^x)e^x\cos(3x+1) - 3\sin(e^x)\sin(3x+1)$$

7. For the function f given below, decompose f into simpler functions in order to find a formula for $f'(x)$:

$$f(x) = (3x + 5)^4\sqrt{x^5 - x} + g(3x + 1) + (g(x))^3,$$

where g is a differentiable function. Your answer will involve g' and g.

$$f = \underbrace{\underbrace{pow_4 \circ (3pow_1 + 5)}_{\text{chain}} \; \underbrace{pow_{\frac{1}{2}} \circ (pow_5 - pow_1)}_{\text{chain}}}_{\text{product}} + \underbrace{g \circ (3pow_1 + 1)}_{\text{chain}} + \underbrace{pow_3 \circ g}_{\text{chain}}$$

So

$$f'(x) = (pow_4 \circ (3pow_1 + 5))'(x) \, pow_{\frac{1}{2}} \circ (pow_5 - pow_1)(x) + pow_4 \circ (3pow_1 + 5)(x) \bullet (pow_{\frac{1}{2}} \circ (pow_5 - pow_1))'(x)$$

$$+ (g \circ (3pow_1 + 1))'(x) + (pow_3 \circ g)'(x)$$

$$= pow'_4 \circ (3pow_1 + 5)(x)\,(3pow_1 + 5)'(x) \, pow_{\frac{1}{2}} \circ (pow_5 - pow_1)(x)$$

$$+ pow_4 \circ (3pow_1 + 5)(x) \bullet (pow'_{\frac{1}{2}} \circ (pow_5 - pow_1)(x))(pow_5 - pow_1)'(x) + g' \circ (3pow_1 + 1)(x)(3pow_1 + 5)'(x)$$

$$+ pow'_3 \circ g(x) \bullet g'(x)$$

$$= 4pow_3 \circ (3pow_1 + 5)(x)\,(3)\, pow_{\frac{1}{2}} \circ (pow_5 - pow_1)(x)$$

$$+ pow_4 \circ (3pow_1 + 5)(x) \cdot \left(\frac{1}{2} pow_{-\frac{1}{2}} \circ (pow_5 - pow_1)(x)\right)(5pow_4(x) - 1) + g' \circ (3pow_1 + 1)(x)(3) + 3pow_2 \circ g(x)g'(x)$$

$$= 12(3x+5)^3\sqrt{x^5 - x} + (3x+5)^4 \cdot \left(\frac{5x^4 - 1}{2\sqrt{x^5 - x}}\right) + 3g'(3x+1) + 3(g(x))^2 g'(x)$$

8. Use Newton's Method to approximate the value $5^{\frac{1}{5}}$. Start with an initial guess of $x_1 = 1$ and apply the method three times.

Take $f(x) = x^5 - 5$. A zero of f is $5^{\frac{1}{5}}$.

Take $x_1 = 1$. The next guess is

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1 - \frac{(1)^5 - 5}{5(1)^4} = 1 + \frac{4}{5} = \frac{9}{5}.$$

The next guess is

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = \frac{9}{5} - \frac{\left(\frac{9}{5}\right)^5 - 5}{5\left(\frac{9}{5}\right)^4} = \frac{9}{5} - \frac{\left(\frac{43424}{3125}\right)}{\left(\frac{6561}{125}\right)} = \frac{251821}{164025}$$

The next guess is

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = \frac{4644239\,310\,705\,401\,965\,739\,580\,529}{3297978\,557\,211\,177\,132\,832\,630\,125}$$

$x_4 = 1.40821$