1. Take f to be a function with the property that

$$\lim_{x \to -1^-} f(x) = 5 = \lim_{x \to -1^+} f(x).$$

Determine the value of $f(-1)$ so that f is continuous at -1.

2. Sketch a function f that is continuous at $x = 3$ but not continuous at $x = 4$.

3. Show that the function f is continuous at 4, where f is given by

$$f(x) = \begin{cases} \dfrac{1 - \cos(x-4)}{x-4} & \text{if } x \neq 4 \\ 2x - 8 & \text{if } x = 4. \end{cases}$$

4. Show that the function f is continuous at 1, where f is given by

$$f(x) = \begin{cases} x^2 + 5 & \text{if } x < 1 \\ 6 & \text{if } x = 1 \\ \log_2(x + 63) & \text{if } x > 1. \end{cases}$$

5. Take f to be the function that is given by

$$f(x) = \frac{1 - \cos(x - 4)}{x - 4}.$$

Determine the maximal domain of f and find a continuous extension of f to all of $\mathbb{R}$.

6. Carefully show that the function f is not continuous at 2, where f is given by

$$f(x) = \begin{cases} \exp_5(x) & \text{if } x \leq 2 \\ 2x + 5 & \text{if } x > 2 \end{cases} \quad \text{where} \quad \exp_5(x) = 5^x.$$

7. Take f and g to be functions that are defined on

$$\mathcal{D}(f) = (-\infty, \infty) \quad \text{and} \quad \mathcal{D}(g) = [-6, \infty)$$

and that are continuous on

$$S_f = (-\infty, 3) \cup (3, 6) \cup (6, \infty) \quad \text{and} \quad S_g = [-6, -2) \cup (-2, 0) \cup (0, 2) \cup (2, \infty),$$

respectively.

a. Determine whether f is defined at $x = 3$.

b. Determine whether f is continuous at $x = 3$ and $x = 9$.

c. Determine whether g is defined at $x = -7$.

d. Determine whether g continuous at $x = -2$ and $x = 2$.

e. Determine the maximal set on which $f + g$ is continuous on.

f. Determine the maximal set on which fg is continuous on.

8. Decompose the function f into sums, products, and quotients of continuous functions to find a set S on which f is continuous, where f is given by

$$f(x) = \frac{\sqrt{10-x}}{x^2-6} + \ln(x) + \sin(x).$$

9. Write the function f as a composite function to determine $\lim_{x \to 4} f(x)$, where f is given by

$$f(x) = \cos\left(\frac{1-\cos(x-4)}{x-4}\right).$$

10. Construct a function that is continuous everywhere except at 1, that is strictly increasing to the left of 1, is asymptotically equal to $-2x^2$ to the left, is strictly decreasing to the right of 1, is right continuous at 1, is asymptotically equal to $-3x^3$, and has the property that

$$\lim_{x\to 1^-} f(x) = 2 \quad \text{and} \quad \lim_{x\to 1^+} f(x) = 3.$$

11. Construct a continuous path c with domain $[0, \infty)$ that describes the position of a particle that moves to the right on the line segment from $(1, 3)$ to $(4, 5)$, is at $(1, 3)$ at time 0, is never at the same point at different time points, that never reaches $(4, 5)$, but that has the property that

$$\lim_{t\to\infty} \|(4, 5) - c(t)\| = 0.$$

12. Take f to be the polynomial that is given by

$$f(x) = x^5 - 4x^3 + 2x - 1.$$

Show that f has at least one real root by using the intermediate value theorem.

13. Use the bisection method to approximate a solution to the equation

$$x^2 = 7$$

to within an error of no greater than $\frac{1}{10}$.