1. Take f to be the function given by

$$f(x) = \begin{cases} \dfrac{x^2 - 3x - 10}{x - 5} & \text{if } x \neq 5 \\ 12 & \text{if } x = 5. \end{cases}$$

Determine $\lim_{x \to 5} f(x)$ and whether the limit depends on $f(5)$.

2. Take f to be a function with the property that $\lim_{x \to 2} f(x) = 7$. Determine $\lim_{x \to 2^-} f(x)$ and $\lim_{x \to 2^+} f(x)$.

3. Show that the following limit does not exist:

$$\lim_{x \to 6} \frac{|x-6|}{x-6}.$$

4. Create a function for which $\lim_{x\to 3^+} f(x)$ and $\lim_{x\to 3^-} f(x)$ exist, but $\lim_{x\to 3} f(x)$ does not exist.

5. Determine the way in which the following limits diverge:

a) $\lim_{x\to 6^+} \frac{1}{(x-6)^3}$

b) $\lim_{x\to 6^-} \frac{1}{(x-6)^3}$

6. Take a, b, c, and d to be functions that are defined in an open interval that contains 2 and

$$\lim_{x\to 2} a(x) = 6, \quad \lim_{x\to 2} b(x) = -1, \quad \lim_{x\to 2} c(x) = 4, \text{and} \quad \lim_{x\to 2} d(x) = 2.$$

Use the limit laws to determine the following limits:

$$\lim_{x\to 2}\left((a(x))^2 b(x) + \frac{(x-3)c(x)}{d(x)+3}\right).$$

7. Determine the following limit and carefully justify your reasoning:

$$\lim_{x\to 3} \frac{\sin(4(x-3))}{6(x-3)}.$$

8. Use the squeeze theorem to determine the following limit: $\lim_{x\to 5} \sqrt{|x-5|}\sin\left(\frac{1}{x-5}\right)$.

9. Take f to be the function given by

$$f(x) = \begin{cases} \dfrac{x^2+x-2}{x-1} & \text{if } x < 1 \\ 3x-1 & \text{if } x \geq 1. \end{cases}$$

a) Determine $\lim_{x\to 1^-} f(x)$.

b) Determine $\lim_{x\to 1^+} f(x)$.

c) Determine whether $\lim_{x\to 1} f(x)$ exists and determine the limit if it exists.

10. Take a to be a real number and f to be the function given by

$$f(x) = \begin{cases} x^2 - 1 & \text{if } x \leq 4 \\ a\log_2(x) + 5 & \text{if } x > 4. \end{cases}$$

a) Determine $\lim_{x \to 4^-} f(x)$.

b) Determine $\lim_{x \to 4^+} f(x)$.

c) Determine a value for a so that $\lim_{x \to 4} f(x)$ exist.

11. Calculate the following:

a) $\lim_{x\to 5} \frac{x-1}{|x-5|}$

b) $\lim_{x\to\infty} \left(\sqrt{16x^2+2x}-4x\right)$

12. Determine all horizontal and vertical asymptotes of the function f that is given by

$$f(x) = \begin{cases} 5^x + 1 & \text{if } x < -2 \\ \dfrac{5x+1}{(x+4)(x-4)} & \text{if } -2 \leq x < 4 \\ \arctan(-x) + \frac{5}{x+2} & \text{if } x \geq 4. \end{cases}$$

13. Take f to be the rational function that is given by

$$f(x) = \frac{x^4(x+4)^6(x-1)^2(x-5)^2}{(1-4x)^3(x+3)^8}.$$

Find a monomial g so that f and g have the same asymptotic behavior at both ∞ and $-\infty$.

14. Identify a path that describes the position in time of a particle that moves along the line segment L with endpoints $(2, 4)$ and $(5, 1)$, has domain equal to $\mathbb{R}$, is at the midpoint of L at time 0, moves only to the left, and reaches all points of L except the endpoints of L.