1. Write the first four terms of $(a_n)$, where for each natural number $n$, $a_n$ is given by $a_n = 5^n$.

2. Show that the sequence $(a_n)$ is increasing and the sequence $(b_n)$ is decreasing, where

a) $a_n = \frac{n+3}{n+5}$

b) $b_n = \frac{n}{n^2+1}$

3. Identify an example of a sequence $(a_n)$ with the following properties:

a) $(a_n)$ is strictly increasing and converges to $2$.

b) $(a_n)$ is strictly decreasing and converges to $\frac{1}{4}$.

4. Use only the Archimedean property of $\mathbb{R}$ to show that for any positive real number $\varepsilon$, there is a natural number $N$ so that if $n$ is greater than $N$, then

$$|a_n - L| < \varepsilon,$$

for the following choices of $(a_n)$ and $L$:

a) $a_n = \frac{9n+2}{3n+4}$ and $L = 3$

b) $a_n = \sqrt{16 + \frac{1}{n}}$ and $L = 4$.

5. Calculate $\lim_{n\to\infty} \sqrt{100 + \frac{1}{n}}$. Carefully justify your reasoning.

6. Take $(a_n)$, $(b_n)$, and $(c_n)$ to be sequences with

$$\lim_{n\to\infty} a_n = 5, \quad \lim_{n\to\infty} b_n = 1, \quad \text{and} \quad \lim_{n\to\infty} c_n = -2.$$

Use the limit laws to compute the following:

$$\lim_{n\to\infty} \frac{(a_n)^3 + 2b_n}{c_n + \frac{1}{n^5}}.$$

7. Use the limit laws to determine the following limits:

a) $\lim_{n\to\infty} \frac{5n}{3n-1}$

b) $\lim_{n\to\infty} \frac{n^2+4n}{n^5+n^2+2}$

8. Take $(a_n)$ and $(b_n)$ to be sequences with

$$\lim_{n\to\infty} a_n = 2 \quad \text{and} \quad \lim_{n\to\infty} \frac{b_n}{a_n} = 18.$$

Carefully justify that $(b_n)$ is convergent and calculate its limit.

9. Calculate $\lim_{n\to\infty} n\left(\sqrt{100+\frac{1}{n}}-10\right)$. Carefully justify your reasoning.

10. Take $a_n$ to be the sequence that is given by

$$a_n = \frac{20n^2+5n\cos(n)}{4n^2+4}.$$

Calculate $\lim_{n\to\infty} a_n$. Carefully justify your reasoning.

11. Determine whether the sequence diverges to either infinity or negative infinity:

a) $a_n = \left(\frac{1}{8}\right)^{-n}$ b) $a_n = \ln(n+1)$ c) $a_n = -n^5$