1. Determine the equation of the line that is tangent to the ellipse E at the point $(8, 12)$, where E is the ellipse that is given by the equation

$$\frac{(x+2)^2}{25} + \frac{(y-4)^2}{16} = 8.$$

① $\frac{(x+2)^2}{25} + \frac{(y-4)^2}{16} = 8$

$\left(\frac{x+2}{5}\right)^2 + \left(\frac{y-4}{4}\right)^2 = 8$

$\left(\frac{1}{5}x + \frac{2}{5}\right)^2 + \left(\frac{1}{4}y - 1\right)^2 = 8$

$\Rightarrow E = (X_5 \circ Y_4)C$

where C is the circle

$(x+\frac{2}{5})^2 + (y-1)^2 = 8$

Find line tangent to C at the point

$Y_{\frac{1}{4}} X_{\frac{1}{5}} (8, 12) = \left(\frac{8}{5}, 3\right)$

② $V = \left(\frac{8}{5}, 3\right) - \left(-\frac{2}{5}, 1\right)$

$= \langle \frac{8}{5} + \frac{2}{5}, 3 - 1 \rangle$

$= \langle 2, 2 \rangle$,

so

$V_\perp = \langle -2, 2 \rangle$

Tangent line to C at $\left(\frac{8}{5}, 3\right)$ is

$y = -(x - \frac{8}{5}) + 3.$

or

$L(x) = -(x - \frac{8}{5}) + 3$

③ Scale C to get E and scale tangent line to C to get tangent line to E.

$4L(\frac{x}{5}) = 4\left(-(\frac{x}{5} - \frac{8}{5}) + 3\right)$

$= -4(\frac{x}{5} - \frac{8}{5}) + 12$

$= -\frac{4}{5}(x-8) + 12$

$\Rightarrow$ Line tangent to E at $(8, 12)$ is

$y = -\frac{4}{5}(x-8) + 12$

2. The line tangent to a quadratic polynomial f at $(2, 5)$ is given by

$$L(x) = 7(x-2) + 5.$$

Determine the line tangent to f^{-1} at $(5, 2)$.

We have

$L(x) = f'(2)(x-2) + 5,$

so $f'(2) = 7$

The reflected line will be tangent to f^{-1} and will have

$(f^{-1})'(5) = \frac{1}{f'(2)} = \frac{1}{7}$

Thus, the line tangent to f^{-1} is

$y = \frac{1}{7}(x-5) + 2.$

3. Find the the equation of the line L tangent to $f(x) = \sqrt{x}$ at $(625, 25)$.

$f(x) = \sqrt{x} = \mathrm{Pow}_{\frac{1}{2}}(x)$, so $f'(x) = \frac{1}{2\sqrt{x}}$ and $f'(625) = \frac{1}{2\sqrt{625}} = \frac{1}{50}$

So the line tangent to f at $(625, 25)$ is

$$L(x) = \frac{1}{50}(x - 625) + 25.$$