1. Graph the following function. Identify the domain, range, and asymptotes of f.

$$f(x) = -5^{x+1} + 2.$$

$f(x) = -5^{x+1} + 2$

$= T_2 \circ S_{-1} \circ \exp_5 \circ T_1$

1. Translate – x
2. Scale
3. Translate

] y

$\exp_5(x) = 5^x$

x	0	1
y	1	5

HA: $y=0$
D: $(-\infty, \infty)$
R: $(0, \infty)$

x transformation
Subtract 1 →

5^{x+1}

x	-1	0
y	1	5

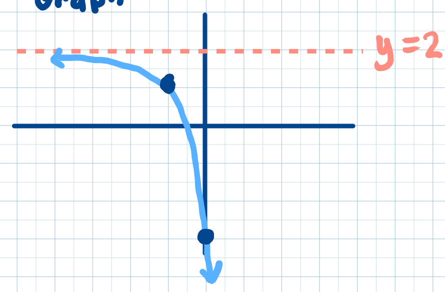
HA: $y=0$
D: $(-\infty,\infty)$
R: $(0,\infty)$

y transformations
2. multiply by -1 →
3. add 2

x	-1	0
y	1	-3

HA: $y=2$
D: $(-\infty,\infty)$
R: $(-\infty, 2)$

Graph



Domain $(-\infty, \infty)$

Range $(-\infty, 2)$

Horizontal Asymptote $y = 2$

2. Compute the following values.

a) $\log_3(243)$

This equals the real number y so that 3 to the y power is 243:

$$3^y = 243.$$

So $\log_3(243) = 5.$

b) $\log_7\left(\frac{1}{49}\right)$

This equals the real number y so that 7 to the y power is $\frac{1}{49}$:

$$7^y = \frac{1}{49}$$

So $\log_7\left(\frac{1}{49}\right) = -2.$

c) $\ln\left(\frac{1}{e}\right)$

This equals the real number y so that e to the y power is $\frac{1}{e}$:

$$e^y = \frac{1}{e}$$

So $\ln\left(\frac{1}{e}\right) = -1.$

d) $\log_{\frac{1}{7}}(343)$

This equals the real number y so that $\frac{1}{7}$ to the y power is 343:

$$\left(\tfrac{1}{7}\right)^y = 343$$

So $\log_{\frac{1}{7}}(343) = -3.$

3. Graph the following function and identify the domain, range, and asymptotes of f:

$$f(x) = \log_{\frac{1}{2}}(-x-2)+2.$$

$f(x) = \log_{\frac{1}{2}}(-x-2)+2$

$= T_2 \circ \log_{\frac{1}{2}} \circ S_{-1} \circ T_{-1}$

1. Translate
2. Scale] x
3. Translate - y

$\log_{\frac{1}{2}}(x)$

x	1	$\frac{1}{2}$
y	0	1

VA: $x=0$
D: $(0,\infty)$
R: $(-\infty,\infty)$

x transformations
1. add 2
2. multiply by -1

$\log_{1/2}(-x-2)$

x	-3	$-\frac{5}{2}$
y	0	1

VA: $x=-2$
D: $(-\infty,-2)$
R: $(-\infty,\infty)$

y transformation
3. add 2

$\log_{\frac{1}{2}}(-x-2)+2$

x	-3	$-\frac{5}{2}$
y	2	3

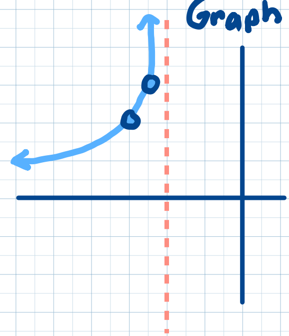
VA: $x=-2$
D: $(-\infty,-2)$
R: $(-\infty,\infty)$

Domain $(-\infty,-2)$

Range $(-\infty,\infty)$

Vertical Asymptote $x=-2$

Graph



4. Solve the following equations.

a) $5^x - 10 = 0$

$5^x - 10 = 0$

$5^x = 10$

$x = \log_5(10)$

or

$x = \dfrac{\ln(10)}{\ln(5)}$

b) $4^x - \left(\frac{1}{16}\right)^{x+1} = 0$

$4^x - \left(\frac{1}{16}\right)^{x+1} = 0$

$4^x = \left(\frac{1}{16}\right)^{x+1}$

$4^x = \left(4^{-2}\right)^{x+1}$ use $\frac{1}{16} = 4^{-2}$

$4^x = 4^{-2x-2}$ use $4^A = 4^B$ iff $A=B$

$x = -2x-2$

$\Rightarrow x = -\frac{2}{3}$

c) $\log_3(-x-1) - \log_3(-x+12) = -2$

Solution must be in domain of $\log_3(-x-1)$ and $\log_3(-x+12)$:

$(-\infty,-1) \cap (-\infty,12) = (-\infty,-1)$

$\log_3(-x-1) - \log_3(-x+12) = -2$ use $\log(A) - \log(B) = \log\left(\frac{A}{B}\right)$

$\log_3\left(\frac{-x-1}{-x+12}\right) = -2$ use $A = \log_b(b^A)$

$\log_3\left(\frac{-x-1}{-x+12}\right) = \log_3(3^{-2})$ use $\log_3(A) = \log_3(B)$ iff $A=B$

$\frac{-x-1}{-x+12} = 3^{-2}$

$\frac{-x-1}{-x+12} = \frac{1}{9}$

$\Rightarrow x = -\frac{21}{8}$ this in $(-\infty,-1)$ so it is a solution

5. Take a and b to be two real numbers so that $\log_5(a) = 5$ and $\log_5(b) = -4$. Compute the following.

a) $\log_5(25ab)$

$\log_5(25ab) = \log_5(25a) + \log_5(b)$ use (I)

$= \log_5(25) + \log_5(a) + \log_5(b)$ use (I)

$= \log_5(5^2) + \log_5(a) + \log_5(b)$

$= 2 + 5 + (-4)$ use (III) and $\log_5(a) = 5$ $\log_5(b) = -4$

$= 3.$

b) $\log_5(\frac{a^3}{5b^4})$

$\log_5(\frac{a^3}{5b^4}) = \log_5(a^3) - \log_5(5b^4)$ use (II)

$= \log_5(a^3) - (\log_5(5) + \log_5(b^4))$ use (I)

$= \log_5(a^3) - \log_5(5) - \log_5(b^4)$

$= 3\log_5(a) - \log_5(5) - 4\log_5(b)$ use (IV)

$= 3 \cdot 5 - 1 - 4 \cdot (-4)$ use (III) and $\log_5(a) = 5$ $\log_5(b) = -4$

$= 15 - 1 + 16$

$= 30.$

logarithm properties:

a, b in $(0,1) \cup (1, \infty)$

$A > 0$

$B > 0$

x real number

I) $\log_b(AB) = \log_b(A) + \log_b(B)$

II) $\log_b(\frac{A}{B}) = \log_b(A) - \log_b(B)$

III) $\log_b(b^x) = x$

IV) $\log_b(a^x) = x\log_b(a)$

6. A quantity A changes according to a linear model for change and

$$\begin{cases} A(0) = 4 \\ A(1) = 13. \end{cases}$$

Identify a formula for $A(t)$.

Linear change is $13 - 4 = 9$, so
$A(t) = 9t + 4$.

7. A quantity A changes according to an exponential model for change and

$$\begin{cases} A(2) = 10 \\ A(5) = 9. \end{cases}$$

Identify a formula for $A(t)$.

Exponential change is $\frac{9}{10}$, so

$$A(t) = 10\left(\frac{9}{10}\right)^{\frac{t-2}{3}}$$

8. A mass of bacteria experiences exponential growth. At time 3 an experimenter has 30 grams of bacteria. At time 7, the mass has grown to a mass of 50.

a) Determine the doubling time of the bacteria.

b) Determine the time that it takes for the amount of the material to increase by a factor of 3.

c) Determine the growth rate of the substance.

Exponential change is $\frac{50}{30}$, so

$$A(t) = 30\left(\frac{50}{30}\right)^{\frac{t-3}{4}}.$$

Rewrite A like this

$$\begin{aligned} A(t) &= 30\left(\frac{50}{30}\right)^{\frac{t-3}{4}} \\ &= 30\left(\frac{50}{30}\right)^{\frac{t}{4}-\frac{3}{4}} \\ &= 30\left(\frac{50}{30}\right)^{\frac{t}{4}}\left(\frac{50}{30}\right)^{-\frac{3}{4}} \\ &= \underbrace{30\left(\frac{50}{30}\right)^{-3/4}}\left(\frac{50}{30}\right)^{\frac{t}{4}} \\ &= A(0)\left(\frac{50}{30}\right)^{\frac{t}{4}} \end{aligned}$$

a) Doubling time T is a real number so that

$$A(t) = A(0)\,2^{\frac{t}{T}}, \text{ so}$$

$$A(0)\left(\frac{50}{30}\right)^{\frac{t}{4}} = A(0)\,2^{\frac{t}{T}}$$

$$\left(\frac{50}{30}\right)^{\frac{t}{4}} = 2^{\frac{t}{T}}$$

$$\Rightarrow \left(\frac{50}{30}\right)^{\frac{1}{4}} = 2^{\frac{1}{T}}$$

$$\frac{1}{4}\ln\left(\frac{50}{30}\right) = \frac{1}{T}\ln(2)$$

$$\frac{T}{4}\ln\left(\frac{50}{30}\right) = \ln(2)$$

$$T = \frac{4\ln(2)}{\ln\left(\frac{50}{30}\right)}$$

b) Time T to increase by factor of 3 is one in which

$$A(t) = A(0)\,3^{\frac{t}{T}}.$$

Similar to part a), so

$$\Rightarrow \left(\frac{50}{30}\right)^{\frac{1}{5}} = 3^{\frac{1}{T}}$$

which implies that

$$T = \frac{5\ln(3)}{\ln\left(\frac{50}{30}\right)}$$

c) growth rate is a positive real number k so that

$$A(t) = A(0)e^{kt}.$$

Use $X = e^{\ln(X)}$ for $X>0$ to get

$$A(t) = A(0)\left(\frac{50}{30}\right)^{\frac{t}{5}} = A(0)e^{\ln\left(\left(\frac{50}{30}\right)^{\frac{t}{5}}\right)} = A(0)e^{\frac{1}{5}\ln\left(\frac{50}{30}\right)t} \Rightarrow k = \frac{1}{5}\ln\left(\frac{50}{30}\right)$$