1. Identify the amplitude, fundamental period and phase shift of the following functions:

$$f(x) = -\sin(\pi x + 3) - 1.$$

$A\sin(B(x-C))$

$|A|$ amplitude

$\frac{2\pi}{|B|}$ fundamental period

$C = \frac{2\pi K}{B}$ K is phase shift

$f(x) = -\sin(\pi(x+\frac{3}{\pi})) - 1$

$A=-1$, $B=\pi$, $C=\frac{3}{\pi}$

$|-1| = 1$ is the amplitude

$\frac{2\pi}{|\pi|} = 2$ is the fundamental period

$\frac{3}{\pi} = \frac{2\pi K}{\pi}$

$\frac{3}{2\pi} = K$ is the phase shift.

2. Take f to be a function with fundamental period equal to $\frac{1}{2}$ and take g to be given by

$$g(x) = 7x + \pi.$$

Determine the fundamental period of $f \circ g$.

The fundamental period of $(f\circ g)(x) = f(g(x)) = f(7x+\pi)$ is

$$\frac{\text{fundamental period of } f}{7} = \frac{\frac{1}{2}}{7} = \frac{1}{14}.$$

3. Graph the following function and identify the fundamental period, amplitude, domain and range.

$$f(x) = -\sin(\pi x + 3) - 1.$$

Recall
$T_h(x) = x + h$
$S_a(x) = ax$

$f(x) = -1 \sin(\pi x + 3) - 1$
$= (T_{-1} \circ S_{-1} \circ \sin \circ S_{\pi} \circ T_3)(x)$

1. Translate x
2. Scale x
x

3. Scale y
4. Translate y
y

x transformations

sin(x)

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y	0	1	0	-1	0

D: $(-\infty, \infty)$
R: $[-1, 1]$

1. Subtract 3
2. Divide by π
$\longrightarrow$

$\sin(\pi x + 3)$

x	$-\frac{3}{\pi}$	$\frac{1}{2} - \frac{3}{\pi}$	$1 - \frac{3}{\pi}$	$\frac{3}{2} - \frac{3}{\pi}$	$2 - \frac{3}{\pi}$
y	0	1	0	-1	0

D: $(-\infty, \infty)$
R: $[-1, 1]$

y transformations

$\sin(\pi x + 3)$

x	$-\frac{3}{\pi}$	$\frac{1}{2} - \frac{3}{\pi}$	$1 - \frac{3}{\pi}$	$\frac{3}{2} - \frac{3}{\pi}$	$2 - \frac{3}{\pi}$
y	0	1	0	-1	0

D: $(-\infty, \infty)$
R: $[-1, 1]$

3. Multiply by -1
4. Subtract 1
$\longrightarrow$

$-\sin(\pi x + 3) - 1$

x	$-\frac{3}{\pi}$	$\frac{1}{2} - \frac{3}{\pi}$	$1 - \frac{3}{\pi}$	$\frac{3}{2} - \frac{3}{\pi}$	$2 - \frac{3}{\pi}$
y	-1	-2	-1	0	-1

D: $(-\infty, \infty)$
R: $[-2, 0]$

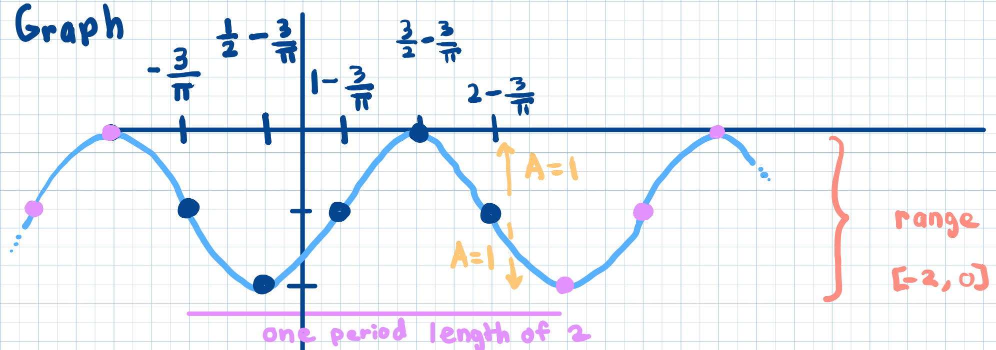
Fundamental Period $\quad \frac{2\pi}{\pi} = 2$

Amplitude $\quad |-1| = 1$

Domain $\quad (-\infty, \infty)$

Range $\quad [-2, 0]$

Graph

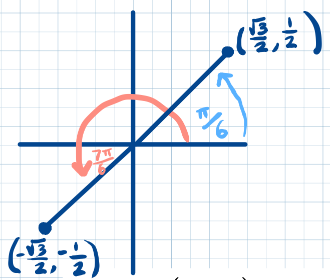


4. Calculate the following.

a) $\cot\left(\frac{7\pi}{6}\right)$

Cotangent is the reciprocal of tangent.

$$\cot\left(\frac{7\pi}{6}\right) = \frac{1}{\tan\left(\frac{7\pi}{6}\right)} = \frac{1}{\left(\frac{1}{\sqrt{3}}\right)} = \sqrt{3}.$$



b) $\csc(\theta)$ given that $\sin(\theta) = -\frac{1}{9}$

Cosecant is the reciprocal of sine.

$$\csc(\theta) = \frac{1}{\sin(\theta)} = \frac{1}{\left(-\frac{1}{9}\right)} = -9.$$

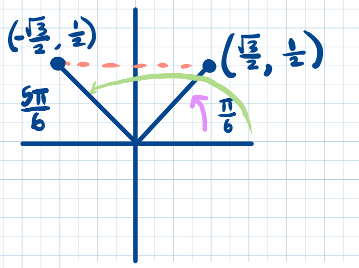
c) $\arccos\left(-\frac{\sqrt{3}}{2}\right)$

arccosine is the inverse of $\cos\big|_{[0,\pi]}$

Identify θ in $[0,\pi]$ so that

$$\cos(\theta) = -\frac{\sqrt{3}}{2}$$

So $\arccos\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$



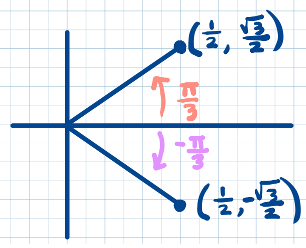
d) $\arcsin\left(-\frac{\sqrt{3}}{2}\right)$

arcsine is the inverse of $\sin\big|_{[-\frac{\pi}{2}, \frac{\pi}{2}]}$

Identify θ in $[-\frac{\pi}{2}, \frac{\pi}{2}]$ so that

$$\sin(\theta) = -\frac{\sqrt{3}}{2}$$

So $\arcsin\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$



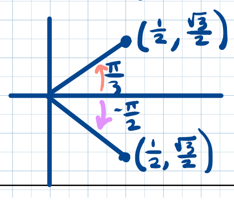
e) $\arctan(-\sqrt{3})$

arctangent is the inverse of $\tan\big|_{(-\frac{\pi}{2}, \frac{\pi}{2})}$

Identify θ in $(-\frac{\pi}{2}, \frac{\pi}{2})$ so that

$$\tan(\theta) = -\sqrt{3}$$

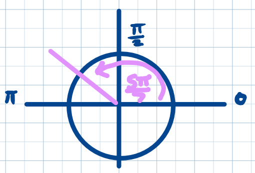
So $\arctan(-\sqrt{3}) = -\frac{\pi}{3}$



5. Calculate the following.

a) $\arccos\left(\cos\left(\frac{5\pi}{7}\right)\right)$

$\arccos(\cos(\theta)) = \theta$
if θ in $[0, \pi]$



So $\arccos\left(\cos\left(\frac{5\pi}{7}\right)\right) = \frac{5\pi}{7}$

b) $\arcsin\left(\sin\left(\frac{3\pi}{4}\right)\right)$

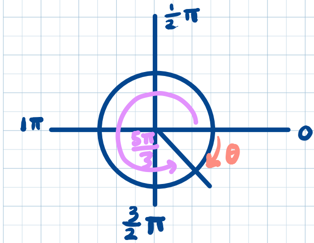
$\arcsin(\sin(\theta)) = \theta$
if θ in $[-\frac{\pi}{2}, \frac{\pi}{2}]$

$\frac{3\pi}{4}$ not in $[-\frac{\pi}{2}, \frac{\pi}{2}]$
but
$\theta = \pi - \frac{3}{4}\pi$
$= \frac{\pi}{4}$
is an angle in $[-\frac{\pi}{2}, \frac{\pi}{2}]$
so that
$\sin(\frac{\pi}{4}) = \sin(\frac{3\pi}{4})$

So $\arcsin(\sin(\frac{3\pi}{4})) = \arcsin(\sin(\frac{\pi}{4})) = \frac{\pi}{4}$

c) $\arctan\left(\tan\left(\frac{5\pi}{3}\right)\right)$

$\arctan(\tan(\theta)) = \theta$
if θ in $(-\frac{\pi}{2}, \frac{\pi}{2})$.



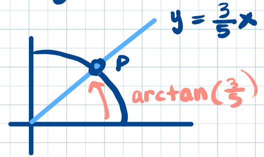
$\frac{5\pi}{3}$ is not in $(-\frac{\pi}{2}, \frac{\pi}{2})$
but
$\theta = \frac{5\pi}{3} - 2\pi$
$\theta = -\frac{\pi}{3}$
is an angle in $(-\frac{\pi}{2}, \frac{\pi}{2})$
so that
$\tan(-\frac{\pi}{3}) = \tan(\frac{5\pi}{3})$

So $\arctan(\tan(\frac{5\pi}{3})) = \arctan(\tan(-\frac{\pi}{3}))$
$= -\frac{\pi}{3}$

d) $\sin\left(\arctan\left(\frac{3}{5}\right)\right)$

arctangent is the inverse of $\tan|_{(-\frac{\pi}{2}, \frac{\pi}{2})}$

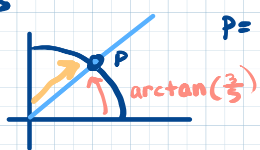
So $\arctan(\frac{3}{5})$ is the angle in $(0, \frac{\pi}{2})$ so that tangent at that angle equals $\frac{3}{5}$:



So the line passing through $(0,0)$ and p has slope $\frac{3}{5}$. A vector that moves points along the line is
$V = \langle 1, \frac{3}{5} \rangle$.

A unit vector in the same direction is
$\hat{V} = \frac{1}{\|V\|} V = \frac{5}{\sqrt{34}} \langle 1, \frac{3}{5} \rangle = \langle \frac{1}{\sqrt{34}}, \frac{3}{\sqrt{34}} \rangle$.

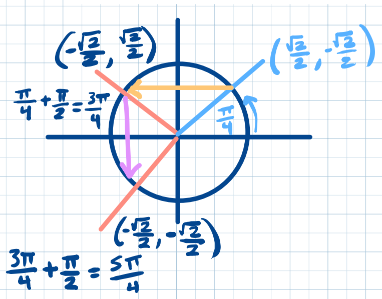
The point p with angle measure $\arctan(3/5)$ is
$p = \hat{V} + (0,0) = \left(\frac{1}{\sqrt{34}}, \frac{3}{\sqrt{34}}\right)$



So $\sin(\arctan(\frac{3}{5})) = \frac{3}{\sqrt{34}}$ or $\frac{3\sqrt{34}}{34}$

6. Solve the following equations.

a) $\cos(x) = -\frac{\sqrt{2}}{2}$ on $[0, 2\pi)$



So solution set is

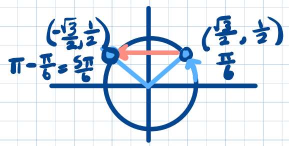
$\left\{\frac{3\pi}{4}, \frac{5\pi}{4}\right\}$

b) $5\sin(x) - \frac{5}{2} = 0$ on $\mathbb{R}$

$5\sin(x) - \frac{5}{2} = 0$

$5\sin(x) = \frac{5}{2}$

$\sin(x) = \frac{1}{2}$



Since sine is 2π periodic, solution set is

$\left\{\frac{\pi}{6} + 2\pi k : k \text{ an integer}\right\} \cup \left\{\frac{5\pi}{6} + 2\pi k : k \text{ an integer}\right\}$

c) $\cos^2(\theta) + \frac{3}{2}\cos(\theta) - 1 = 0$ on $\mathbb{R}$

Set $u = \cos(\theta)$ so $u^2 = \cos^2(\theta)$.

The equation can be rewritten as

$u^2 + \frac{3}{2}u - 1 = 0.$

Solve this quadratic equation:

$u^2 + \frac{3}{2}u - 1 = 0$

$(u + \frac{3}{4})^2 - \frac{25}{16} = 0$

$(u + \frac{3}{4})^2 = \frac{25}{16}$

$\Rightarrow u + \frac{3}{4} = \frac{5}{4}$ or $u + \frac{3}{4} = -\frac{5}{4}$

$u = \frac{1}{2}$ or $u = -2$

$\cos(\theta) = \frac{1}{2}$ has solution $\quad \cos(\theta) = -2$ No solution

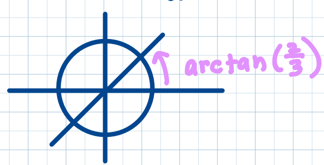
$\Rightarrow$ Solution set is $\left\{\frac{\pi}{3} + 2\pi k : k \text{ an integer}\right\} \cup \left\{\frac{5\pi}{3} + 2\pi k : k \text{ an integer}\right\}$

d) $\tan(x) - \frac{2}{3} = 0$ on $\mathbb{R}$

$\tan(x) - \frac{2}{3} = 0$

$\tan(x) = \frac{2}{3}$

$x = \arctan(\frac{2}{3})$



Since tangent is π periodic, solution set is $\left\{\arctan(\frac{2}{3}) + \pi k : k \text{ an integer}\right\}$

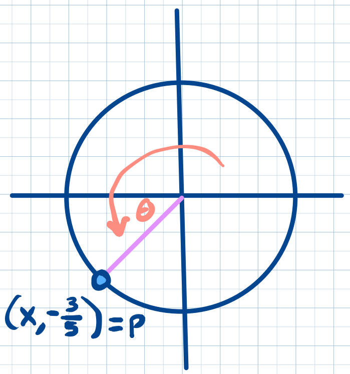
e) $\cos(x) = -1.1$

Range of cosine is $[-1, 1]$,

so this equation has no solution.

7. A point $\left(x, -\frac{3}{5}\right)$ on the unit circle corresponds to an angle θ in quadrant III. Calculate

$$\sin(\theta), \quad \cos(\theta), \quad \tan(\theta), \quad \csc(\theta), \quad \cot(\theta), \quad \text{and} \quad \sec(\theta).$$



P is on the unit circle, so

$$x^2 + \left(-\frac{3}{5}\right)^2 = 1$$

$$x^2 + \frac{9}{25} = 1$$

$$x^2 = 1 - \frac{9}{25}$$

$$x^2 = \frac{16}{25}$$

$$\Rightarrow x = -\frac{4}{5}$$

$\sin(\theta) = -\frac{3}{5}$ $\qquad$ $\csc(\theta) = -\frac{5}{3}$

$\cos(\theta) = -\frac{4}{5}$ $\qquad$ $\sec(\theta) = -\frac{5}{4}$

$\tan(\theta) = \frac{3}{4}$ $\qquad$ $\cot(\theta) = \frac{4}{3}$