1. Determine which of the following functions are even, odd or neither:

a) $f(x) = x^2 + \cos(x)$

$f(-x) = (-x)^2 + \cos(-x)$
$= x^2 + \cos(x).$
This is f, meaning that
$f(-x) = f(x)$,
So f is even.

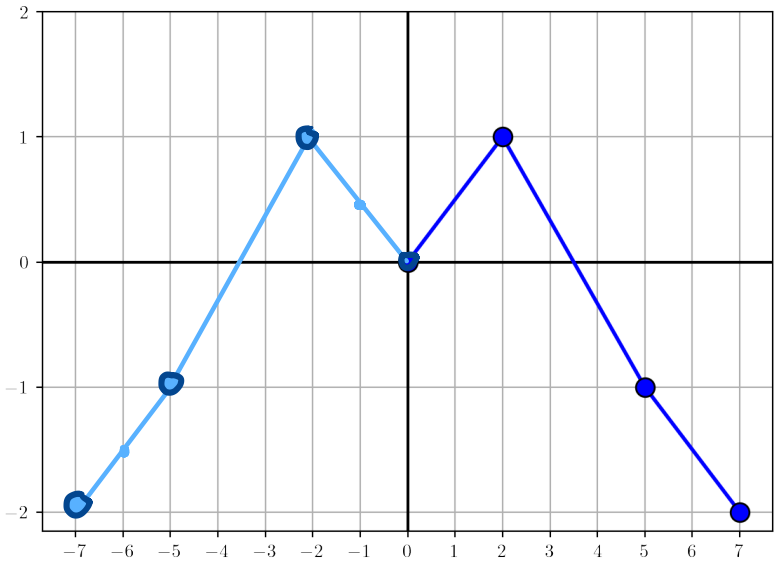
b) $f(x) = x^2 + |x| + x$

$f(-x) = (-x)^2 + |-x| + (-x)$
$= x^2 + x - x.$
This is neither f nor $-f$.
Example:
$f(1) = 1^2 + |1| + 1 = 3,$
$f(-1) = (-1)^2 + |-1| - 1 = 2$
So $f(1) \neq f(-1)$
nor
$f(-1) \neq -f(1)$
so f is neither even nor odd.

c) $f(x) = x^4 \sin(x)$

$f(-x) = (-x)^4 \sin(-x)$
$= x^4(-\sin(x))$
$= -x^4 \sin(x)$
$= -f(x).$
This is $-f$, meaning that
$f(-x) = -f(x)$,
So f is odd.

2. Take f to be a function that is even. Part of its graph is shown below. Sketch what f looks like for values of x that are negative.



f is even means
$f(x) = f(-x)$ for all x in $D(f)$.

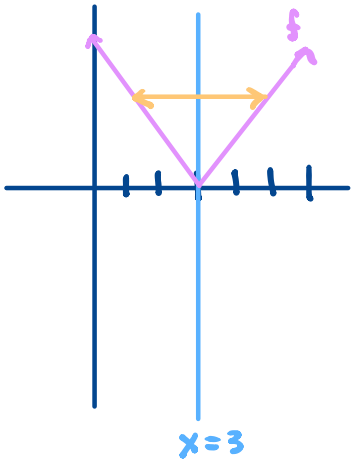
3. Take f to be the function that is given by

$$f(x) = |x - 3|.$$

Determine a vertical line L so that reflection of f across L is equal to f.

The reflection of f across x=3 is f.

Visual



4. Take f to be a function that is given by

$$f(x) = |x - 3|.$$

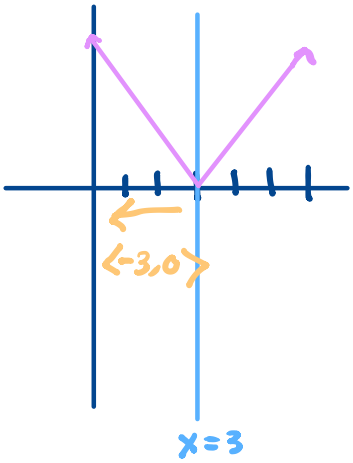
Determine a vector $\langle a, 0\rangle$ so that $\langle a, 0\rangle + f$ is an even function.

Take $V = \langle -3, 0\rangle$. Then,

$$\langle -3,0\rangle + f = \langle -3,0\rangle + \{(x, |x-3|) : x \in \mathbb{R}\}$$
$$= \{(x-3, |x-3|) : x \in \mathbb{R}\} \quad \text{Take } X = x-3$$
$$= \{(X, |X|) : X - 3 \in \mathbb{R}\}$$
$$= g$$

Where $g(x) = |x|$. This function is even.

Visual

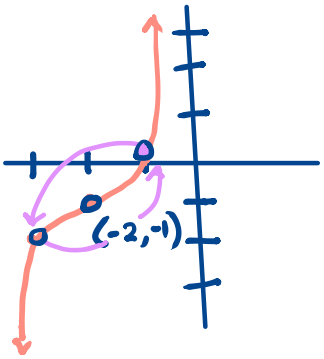


5. Take f to be a function that is given by

$$f(x) = (x+2)|x+2| - 1.$$

Determine a point p so that rotation around p by half a circle equals f.

Rotating f around (-2,-1) by half a circle is f.



6. Take f to be a function that is given by

$$f(x) = (x+2)|x+2| - 1.$$

Determine a vector V so that $V + f$ is an odd function.

Take $V = \langle 2, 1 \rangle$. Then,

$$V + f = \langle 2,1 \rangle + \{(x, (x+2)|x+2| - 1) : x \in \mathbb{R}\}$$

$$= \{(x+2, (x+2)|x+2|) : x \in \mathbb{R}\} \quad X = x+2$$

$$= \{(X, X|X|) : X - 2 \in \mathbb{R}\}$$

$$= g$$

Where $g(x) = x|x|$. This function is odd.