1. Determine the equation of the line that is tangent to the circle C at the point $(-1, 2)$, where C is the circle that is given by the equation

$$(x+2)^2 + (y-4)^2 = 5.$$

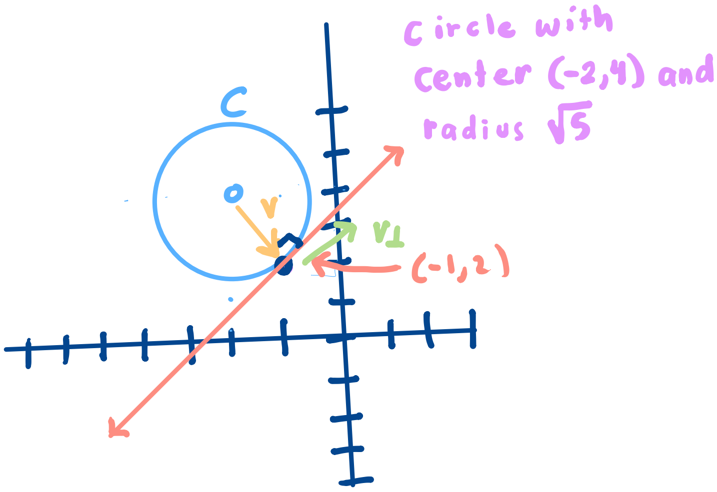
$V = (-1,2) - (-2,4)$
$= \langle 1, -2 \rangle.$

So

$V_\perp = \langle 2, 1 \rangle$

The line L tangent to C at $(-1,2)$ is

$$y = \frac{1}{2}(x+1) + 2$$

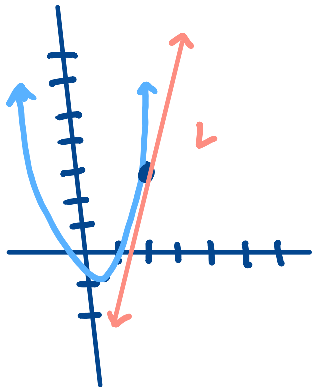


2. Take f to be the quadratic function and L to be the line that are given by

$$f(x) = 3x^2 - 5x + 1 \quad \text{and} \quad L(x) = mx + b.$$

Identify a quadratic equation that determines m so that L is tangent to f at $(2, 3)$.

L must cross at $(2,3)$, so:



$$L(x) = m(x-2) + 3$$
$$= mx - 2m + 3.$$

L is tangent to f if and only if L intersects f only at $(2,3)$:

$$(f - L)(x) = 3x^2 - 5x + 1 - (mx - 2m + 3)$$
$$= 3x^2 - 5x + 1 - mx + 2m - 3.$$
$$= 3x^2 + (-5-m)x + 2m - 2.$$

The function $f - L$ must have only one root, so its discriminant must be zero:

$$\Delta(f-L) = (-5-m)^2 - 4(3)(2m-2)$$
$$= m^2 + 10m + 25 - 24m + 24$$
$$= m^2 - 14m + 49.$$

So equation is

$$m^2 - 14m + 49 = 0.$$

3. Find the the equation of the line L tangent to $f(x) = 3x^2 - 5x + 1$ at $(2, 3)$.

Continue from previous part:

$$m^2 - 14m + 49 = 0$$
$$(m - 7)^2 = 0$$
$$\Rightarrow m = 7.$$

So equation of line tangent to f at (2,3) is

$$y = 7(x-2) + 3$$

or

$$y = 7x - 11.$$

4. Determine the equation of the line L tangent to $f(x) = 10x^3 - 10x + 5$ at $(1, 5)$.

Expand f around $x=1$:

$$
\begin{aligned}
f(x) &= f(x-1+1) \\
&= 10(x-1+1)^3 - 10(x-1+1) + 5 \\
&= 10\left((x-1)^3 + 3(x-1)^2 + 3(x-1) + 1\right) - 10(x-1+1) + 5 \\
&= 10(x-1)^3 + 30(x-1)^2 + 30(x-1) + 10 - 10(x-1) - 10 + 5 \\
&= 20(x-1) + 5 + 10(x-1)^3 + 30(x-1)^2 \\
&= 20(x-1) + 5 + (x-1)^2(10(x-1) + 30)
\end{aligned}
$$

Line tangent to f at $(1,5)$ is

$$y = 20(x-1) + 5.$$