1. Determine whether the following rational function in simplest form:

$$f(x) = \frac{x(x-3)^2}{x^2(x+3)}.$$

Simplest form means numerator or denominator do not share zeros

numerator: $x=0$, $x=3$

denominator: $x=0$, $x=-3$

not in simplest form

2. Find the zeros and poles of the given function and list the order or multiplicity as well:

$$f(x) = \frac{(x+2)^2(x-2)}{x^3(x+4)}.$$

Zeros of f are zeros of numerator:

$(x+2)^2(x-2)=0$

zeros	-2	2
order	2	1

Poles of f are zeros of denominator:

$x^3(x+4)=0$

Poles	0	-4
order	3	1

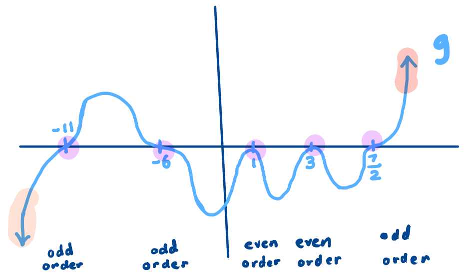
3. For the following function, list the poles of the function and the order of each pole, the vertical asymptotes of f, the horizontal asymptote of f, sketch the denominator of f and then use inversion to sketch the function:

$$f(x) = \frac{1}{20(x+11)^5(x+6)^3(x-1)^2(x-3)^4(2x-7)^5}.$$

Graph denominator $g(x) = 20(x+11)^5(x+6)^3(x-1)^2(x-3)^4(2x-7)^5$

asymptotic behavior: $640x^{19}$

Zeros

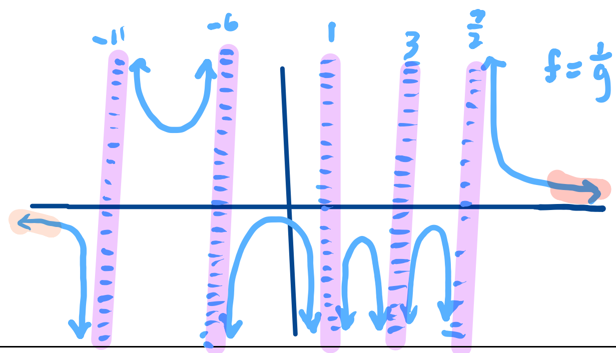


Use y-axis inversion

Zeros → VA

"large" → "small"

"small" → "large"



4. List and classify all asymptotes of

$$f(x) = \frac{x^2 - 4}{(x+9)^2(x-6)}.$$

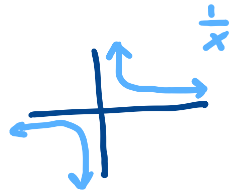
Vertical asymptotes are poles of f

$x = -9, \; x = 6$

Horizontal asymptotes

f behaviors like

$$\frac{\text{leading term of numerator}}{\text{leading term of denominator}} = \frac{x^2}{x^3} = \frac{1}{x}$$



So f has a horizontal asymptote of $y = 0$.