1. Take f to be the function that is given by

$$f(x) = 6(x-2)^2 - 15.$$

a) Determine the axis of symmetry and vertex of f.

axis of symmetry: $x = 2$

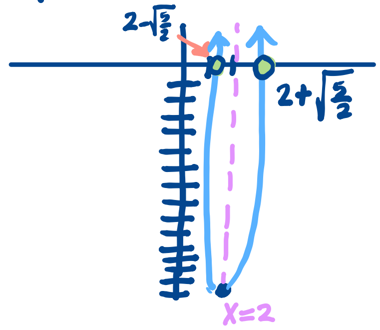
vertex: $(2,-15)$

b) Find all roots (x-intercepts) of f.

$$f(x) = 0$$
$$6(x-2)^2 - 15 = 0$$
$$6(x-2)^2 = 15$$
$$(x-2)^2 = \frac{15}{6}$$
$$(x-2)^2 = \frac{5}{2}$$
$$x-2 = \pm\sqrt{\frac{5}{2}}$$
$$\Rightarrow x = 2+\sqrt{\frac{5}{2}} \text{ or } 2-\sqrt{\frac{5}{2}}$$

c) Sketch f.

$A=6$ is positive, so parabola opens up at vertex $(2,-15)$.



d) Solve $f > 0$.

Use graph to obtain that $f > 0$ on $\left(-\infty, 2-\sqrt{\frac{5}{2}}\right) \cup \left(2+\sqrt{\frac{5}{2}}, \infty\right)$.

2. Find and classify the extremal points of f, where

$$f(x) = -10x^2 + 10x + \frac{1}{2}.$$

① Determine A, h, K so that

$$f(x) = A(x-h)^2 + K \quad \text{expand}$$
$$-10x^2 + 10x + \frac{1}{2} = A(x^2 - 2hx + h^2) + K$$
$$-10x^2 + 10x + \frac{1}{2} = Ax^2 - 2Ahx + Ah^2 + K$$

Solve
$$\begin{cases} -10 = A & (\mathrm{I}) \\ 10 = -2Ah & (\mathrm{II}) \\ \frac{1}{2} = Ah^2 + K & (\mathrm{III}) \end{cases}$$

ⓐ (I) tells us that $A = -10$

ⓑ Use (II) to get
$$10 = -2Ah$$
$$10 = -2(-10)h$$
$$10 = 20h$$
$$\frac{1}{2} = h$$

ⓒ Use (III) to get
$$\frac{1}{2} = Ah^2 + K$$
$$\frac{1}{2} = -10\left(\frac{1}{2}\right)^2 + K$$
$$\frac{1}{2} = -\frac{5}{2} + K$$
$$\frac{1}{2} + \frac{5}{2} = K$$
$$3 = K$$

② Use $A=-10$, $h = \frac{1}{2}$, $K = 3$ to rewrite f like this:

$$f(x) = -10\left(x - \frac{1}{2}\right)^2 + 3.$$

Because $A = -10$ is negative the parabola opens down at vertex $\left(\frac{1}{2}, 3\right)$.

Therefore, f has a local and global max at $x = \frac{1}{2}$. The max value is $y = 3$. There is no global min nor local min.

3. Find polynomials q and r so that $\frac{r(x)}{x-4}$ is a proper fraction and

$$5x^3 + 10x^2 - 5 = q(x)(x-4) + r(x).$$

① $\frac{r(x)}{x-4}$ Proper means r must be one degree less than x−4. So r must be a constant polynomial

$5x^3+10x^2-5$ is degree 3 so $q(x)(x-4)$ must be a degree 3 Polynomial. Thus q must be a degree 2 Polynomial.

$$5x^3+10x^2-5 = q(x)(x-4) + r(x)$$

$$5x^3+10x^2-5 = (Ax^2+Bx+C)(x-4) + D$$

expand and combine like terms

$$5x^3+10x^2+0x-5 = Ax^3+(-4A+B)x^2+(C-4B)x+(D-4C)$$

solve

$$\begin{cases} 5 = A \\ 10 = -4A+B \\ 0 = C-4B \\ -5 = D-4C \end{cases} \Rightarrow A=5,\ B=30,\ C=120,\ D=475$$

② So

$q(x) = 5x^2+30x+120$

$r(x) = 475.$

Can check this is correct by verifying that $5x^3+10x^2-5$ equals $q(x)(x-4)+r(x)$.

4. Write $4x^3 - 32x^2 - 212x + 240$ as a product of linear factors given that $x = -5$ is a root or zero.

① Because $x=-5$ is a root, $x+5$ divides $4x^3-32x^2-212x+240$.

So there is a polynomial q so that $4x^3-32x^2-212x+240 = q(x)(x+5)$.

Need $q(x)(x+5)$ to be degree 3 so q needs to be degree 2.

So,

$$4x^3-32x^2-212x+240 = q(x)(x+5)$$
$$= (Ax^2+Bx+C)(x+5)$$

$$4x^3-32x^2-212x+240 = Ax^3+(5A+B)x^2+(5B+C)x+5C$$

Solve

$$\begin{cases} 4 = A \\ -32 = 5A+B \\ -212 = 5B+C \\ 240 = 5C \end{cases} \Rightarrow A=4,\ B=-52,\ C=48$$

② Use A, B and C to get

$$\begin{aligned} 4x^3-32x^2-212x+240 &= q(x)(x+5) \\ &= (4x^2-52x+48)(x+5) \\ &= 4(x^2-13x+12)(x+5) \\ &= \underbrace{4(x-12)(x-1)(x+5)}_{\text{answer}} \end{aligned}$$

5. Take f to be the polynomial function that is given by

$$f(x) = 20(x+11)^5(x+6)^3(x-1)^2(x-3)^4(2x-7)^5.$$

List the zeros of f together with their orders.

Zeros	-11	-6	1	3	$\frac{7}{2}$
Orders	5	3	2	4	5

6. Determine the leading term of the polynomial f that is given by

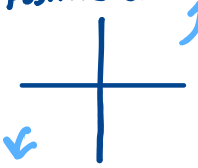
$$f(x) = 20(x+11)^5(x+6)^3(x-1)^2(x-3)^4(2x-7)^5.$$

Describe the asymptotic behavior of f.

Determine the leading term to determine asymptotic behavior:

$$20 \cdot x^5 \cdot x^3 \cdot x^2 \cdot x^4 \cdot (2x)^5$$
$$= 20 \cdot 2^5 x^{5+3+2+4+5}$$
$$= 640x^{19}$$

odd power
positive coefficient



f is asymptoticaly equal to $640x^{19}$ will look like this

7. Sketch the polynomial f, where

$$f(x) = 20(x+11)^5(x+6)^3(x-1)^2(x-3)^4(2x-7)^5.$$

Use asymptotic behavior, zeros and orders to get