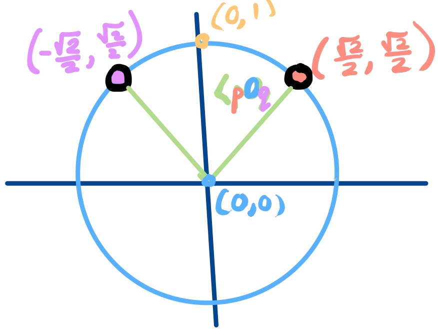
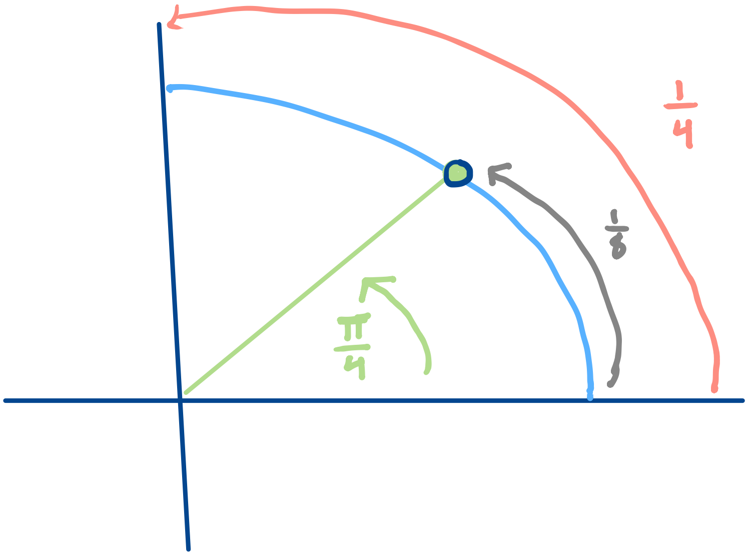
1. Calculate $\angle pOq$ where $p = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ and $q = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

Visual



$$\angle pOq = p^{-1} * q$$
$$= \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right) * \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$
$$= \left(\frac{\sqrt{2}}{2} \cdot \left(-\frac{\sqrt{2}}{2}\right) - \left(-\frac{\sqrt{2}}{2}\right) \cdot \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \cdot \left(-\frac{\sqrt{2}}{2}\right)\right)$$
$$= \left(-\frac{2}{4} + \frac{2}{4}, \frac{2}{4} + \frac{2}{4}\right)$$
$$= (0, 1) \text{ corresponds to } 90°.$$

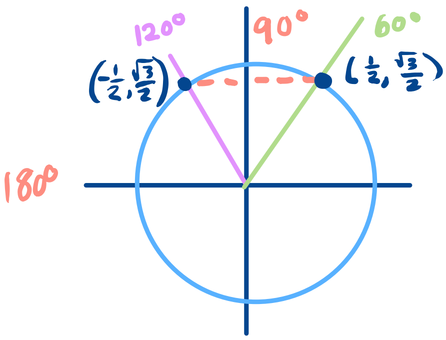
2. Determine the fraction of the circle of the angle whose radian measure is $\frac{\pi}{4}$.



$$\frac{\frac{\pi}{4}}{2\pi} = \frac{\pi}{8\pi} = \frac{1}{8}$$

3. Calculate sine, cosine, and tangent at the following angles $\theta = 120°$, $\theta = \frac{7\pi}{6}$.

$120°$

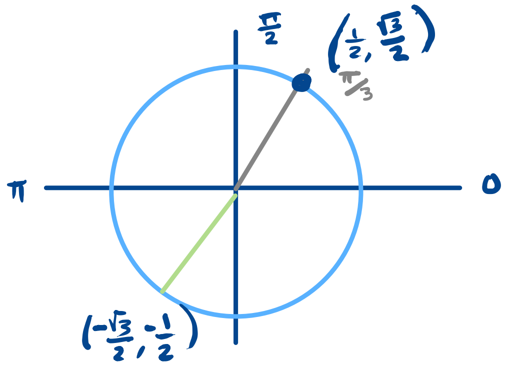


$\sin(120°) = \frac{\sqrt{3}}{2}$

$\cos(120°) = -\frac{1}{2}$

$\tan(120°) = -\sqrt{3}$

$\frac{7\pi}{6}$

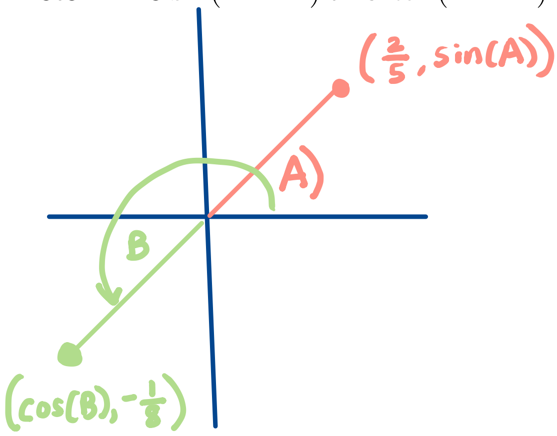


$\sin\left(\frac{7\pi}{6}\right) = -\frac{1}{2}$

$\cos\left(\frac{7\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

$\tan\left(\frac{7\pi}{6}\right) = \frac{1}{\sqrt{3}}$

4. There are angles A in quadrant I and B in quadrant III so that $\cos(A) = \frac{2}{5}$ and $\sin(B) = -\frac{1}{8}$. Determine $\sin(A-B)$ and $\tan(A-B)$.



Need $\sin(A)$ and $\cos(B)$:

$\left(\frac{2}{5}\right)^2 + \sin^2(A) = 1$

$\frac{4}{25} + \sin^2(A) = 1$

$\sin^2(A) = 1 - \frac{4}{25}$

$\sin^2(A) = \frac{21}{25}$

$\Rightarrow \sin(A) = \frac{\sqrt{21}}{5}$

$\cos^2(B) + \left(-\frac{1}{8}\right)^2 = 1$

$\cos^2(B) + \frac{1}{64} = 1$

$\cos^2(B) = 1 - \frac{1}{64}$

$\cos^2(B) = \frac{63}{64}$

$\Rightarrow \cos(B) = -\frac{\sqrt{63}}{8}$

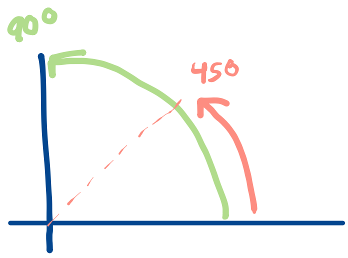
$$\sin(A-B) = \cos(A)\sin(B) - \sin(A)\cos(B)$$
$$= \frac{2}{5} \cdot \left(-\frac{1}{8}\right) - \frac{\sqrt{21}}{5} \cdot \left(-\frac{\sqrt{63}}{8}\right)$$
$$= \frac{-2 + \sqrt{1323}}{40}$$

$$\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$
$$= \frac{2}{5} \cdot \left(-\frac{\sqrt{63}}{8}\right) + \frac{\sqrt{21}}{5} \cdot \left(-\frac{1}{8}\right)$$
$$= \frac{-2\sqrt{63} - \sqrt{21}}{40}$$

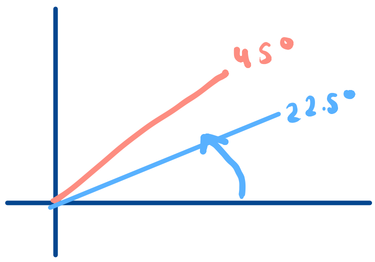
$$\tan(A-B) = \frac{\sin(A-B)}{\cos(A-B)} = \frac{-2+\sqrt{1323}}{-2\sqrt{63} - \sqrt{21}}$$

5. Use the half angle formula to determine $\cos(45°)$ and $\cos(22.5°)$.

$$\cos(45°) = \cos\left(\frac{90°}{2}\right) = \sqrt{\frac{1+\cos(90°)}{2}} = \sqrt{\frac{1+0}{2}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}} \text{ or } \frac{\sqrt{2}}{2}$$



$$\cos(22.5°) = \cos\left(\frac{45°}{2}\right) = \sqrt{\frac{1+\cos(45°)}{2}} = \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{\frac{2+\sqrt{2}}{2}}{2}} = \sqrt{\frac{2+\sqrt{2}}{4}} = \frac{\sqrt{2+\sqrt{2}}}{2}$$



6. Determine $R_\theta(1,2)$.

$$R_\theta(1,2) = (\cos(\theta), \sin(\theta)) * (1,2)$$
$$= (\cos(\theta) - 2\sin(\theta), 2\cos(\theta) + \sin(\theta))$$

Rotate $(1,2)$ by angle θ