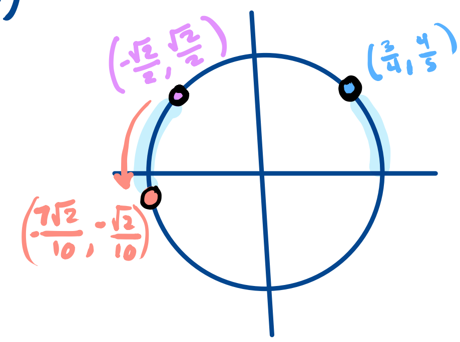
1. Calculate $\left(\frac{3}{5}, \frac{4}{5}\right) \star \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

visual

$$\left(\frac{3}{5}, \frac{4}{5}\right) \star \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) = \left(\frac{3}{5}\cdot\left(-\frac{\sqrt{2}}{2}\right) - \frac{4}{5}\cdot\frac{\sqrt{2}}{2}, \frac{3}{5}\cdot\frac{\sqrt{2}}{2} + \frac{4}{5}\cdot\left(-\frac{\sqrt{2}}{2}\right)\right)$$

$$= \left(-\frac{3\sqrt{2}}{10} - \frac{4\sqrt{2}}{10}, \frac{3\sqrt{2}}{10} - \frac{4\sqrt{2}}{10}\right)$$

$$= \left(-\frac{7\sqrt{2}}{10}, -\frac{\sqrt{2}}{10}\right)$$



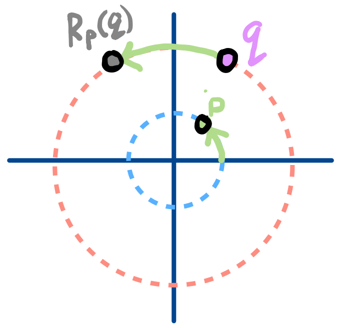
2. Let $p = \left(\frac{1}{4}, \frac{\sqrt{15}}{4}\right)$ on the unit circle. Let $q = (1, 4)$. Calculate $R_p(q)$. State the geometric or visual meaning of this answer.

$$R_p(q) = \left(\frac{1}{4}, \frac{\sqrt{15}}{4}\right) \star (1, 4)$$

$$= \left(\frac{1}{4}\cdot 1 - \frac{\sqrt{15}}{4}\cdot 4, \frac{1}{4}\cdot 4 + \frac{\sqrt{15}}{4}\cdot 1\right)$$

$$= \left(\frac{1 - 4\sqrt{15}}{4}, \frac{4 + \sqrt{15}}{4}\right)$$

This point is the result after rotating q by p around the origin (0,0).



3. Let $p = \left(\frac{1}{4}, \frac{\sqrt{15}}{4}\right)$ and $q = \left(-\frac{1}{4}, -\frac{\sqrt{15}}{4}\right)$. Find a point r so that $p \star r = q$. State the geometric or visual meaning of this answer.

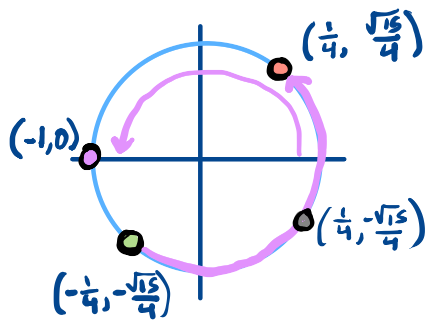
$$r = p^{-1} \star q$$

$$= \left(\frac{1}{4}, -\frac{\sqrt{15}}{4}\right) \star \left(-\frac{1}{4}, -\frac{\sqrt{15}}{4}\right)$$

$$= \left(\frac{1}{4}\cdot\left(-\frac{1}{4}\right) - \left(-\frac{\sqrt{15}}{4}\right)\cdot\left(-\frac{\sqrt{15}}{4}\right), \frac{1}{4}\cdot\left(-\frac{\sqrt{15}}{4}\right) + \left(-\frac{\sqrt{15}}{4}\right)\cdot\left(-\frac{1}{4}\right)\right)$$

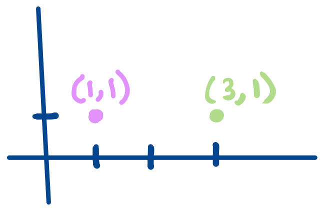
$$= \left(-\frac{1}{16} - \frac{15}{16}, -\frac{\sqrt{15}}{16} + \frac{\sqrt{15}}{16}\right)$$

$$= (-1, 0)$$



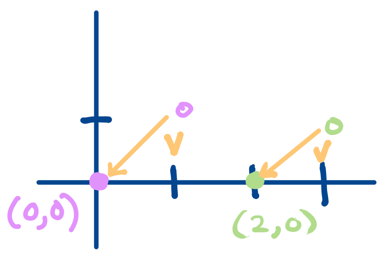
r is the point need so that p goes to q.

4. Rotate the point $(3, 1)$ around the point $(1, 1)$ by the angle $a = \left(\frac{1}{3}, \frac{\sqrt{8}}{3}\right)$.



Translate so center $(1,1)$ is at origin $(0,0)$.

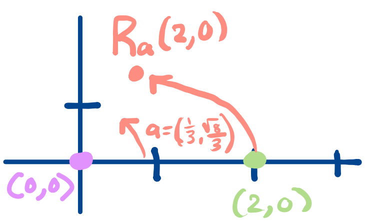
$$V = \langle -1, -1 \rangle$$



$$V + (1,1) = (0,0)$$

$$V + (3,1) = (2,0)$$

Rotate by $a = \left(\frac{1}{3}, \frac{\sqrt{8}}{3}\right)$

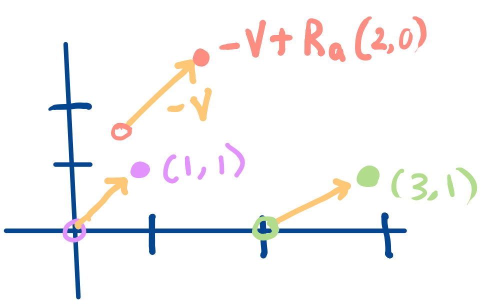


$$R_a(2,0) = \left(\frac{1}{3}, \frac{\sqrt{8}}{3}\right) * (2,0)$$

$$= \left(\frac{1}{3} \cdot 2 - \frac{\sqrt{8}}{3} \cdot 0, \frac{1}{3} \cdot 0 + \frac{\sqrt{8}}{3} \cdot 2\right)$$

$$= \left(\frac{2}{3}, \frac{2\sqrt{8}}{3}\right)$$

Translate by $(0,0)$ to the original center $(1,1)$



$$-V + R_a(2,0) = \langle 1, 1 \rangle + \left(\frac{2}{3}, \frac{2\sqrt{8}}{3}\right)$$

$$= \left(1 + \frac{2}{3}, 1 + \frac{2\sqrt{8}}{3}\right)$$

$$= \left(\frac{5}{3}, \frac{3 + 2\sqrt{8}}{3}\right)$$

final answer