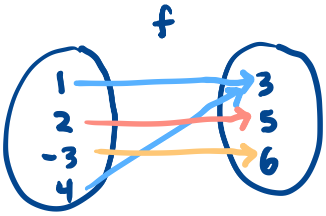
1. Determine whether the function $f = \{(1,3),(2,5),(-3,6),(4,3)\}$ is invertible. If not, remove points so that it is. In either case, write out the inverse.

① Not invertible because of (1,3) and (4,3).



② Remove one of those points:

$f^* = \{(1,3),(2,5),(-3,6)\}$ is invertible and

$(f^*)^{-1} = \{(3,1),(5,2),(6,-3)\}$

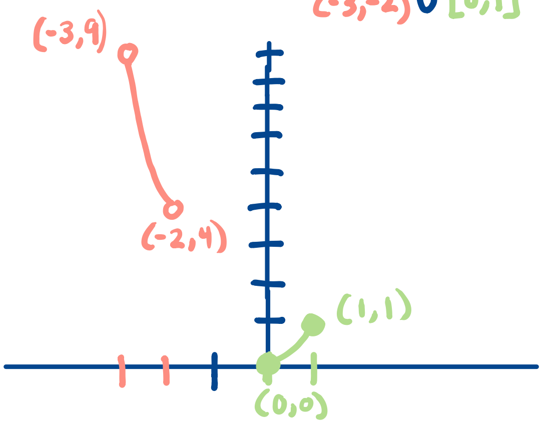
2. Take f to be an invertible function with domain $(4,10]$ and range $(-4,3)$. Determine the domain and range of the inverse of f.

$D(f) = (4,10]$ $\overset{f}{(x,y)} \Leftrightarrow \overset{f^{-1}}{(y,x)}$ $D(f^{-1}) = (-4,3)$ domain of f^{-1}

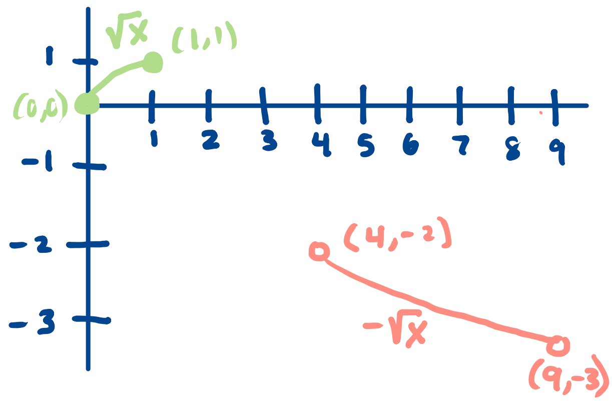
$R(f) = (-4,3)$ $R(f^{-1}) = (4,10]$ range of f^{-1}

3. Sketch $f|_{(-3,-2)\cup[0,1]}$ where $f(x) = x^2$. Then sketch the inverse of $f|_{(-3,-2)\cup[0,1]}$ and write a formula for it.

Sketch of $f|_{(-3,-2)\cup[0,1]}$



Sketch of $\left(f|_{(-3,-2)\cup[0,1]}\right)^{-1}$



$$f^{-1}(x) = \begin{cases} \sqrt{x} & \text{if } 0 \le x \le 1 \\ -\sqrt{x} & \text{if } 4 < x < 9 \end{cases}$$

4. Find the inverse of $f(x) = 3(x-2)^3 + 1$. Determine the domain and range of f and its inverse.

① Rewrite in terms of x

$$x = 3(y-2)^3 + 1$$
$$x+1 = 3(y-2)^3$$
$$\frac{x+1}{3} = (y-2)^3$$
$$\sqrt[3]{\frac{x+1}{3}} = y-2$$
$$\sqrt[3]{\frac{x+1}{3}} + 2 = y$$

② Inverse is

$$f^{-1}(x) = \sqrt[3]{\frac{x+1}{3}} + 2$$

$D(f) = (-\infty, \infty)$ $\quad D(f^{-1}) = (-\infty, \infty)$

$R(f) = (-\infty, \infty)$ $\quad R(f^{-1}) = (-\infty, \infty)$

5. Find the inverse of $f(x) = \frac{x-3}{2x+4}$. Determine the domain and range of f and its inverse.

① Rewrite in terms of x

$$x = \frac{y-3}{2y+4}$$
$$x(2y+4) = y-3$$
$$2xy + 4x = y - 3$$
$$2xy - y = -4x - 3$$
$$y(2x-1) = -4x-3$$
$$y = \frac{-4x-3}{2x-1}$$

② Inverse is

$$f^{-1}(x) = \frac{-4x-3}{2x-1}$$

$f(x) = \frac{x-3}{2x+4}$ $\quad f^{-1}(x) = \frac{-4x-3}{2x-1}$

$D(f) = (-\infty, -2) \cup (-2, \infty)$ $\quad D(f^{-1}) = (-\infty, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$

$R(f^{-1}) = (-\infty, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$ $\quad R(f^{-1}) = (-\infty, -2) \cup (-2, \infty)$

6. Find the inverse of $f(x) = \sqrt{x-2} + 1$. Determine the domain and range of f and its inverse.

① Rewrite in terms of x

$$x = \sqrt{y-2} + 1$$
$$x - 1 = \sqrt{y-2}$$
$$(x-1)^2 = y-2$$
$$(x-1)^2 + 2 = y$$

② Inverse is

$$f^{-1}(x) = (x-1)^2 + 2$$

not invertible unless there is a restriction on domain

$f(x) = \sqrt{x-2} + 1$ $\quad f^{-1}(x) = (x-1)^2 + 2$

$D(f) = [2, \infty)$ $\quad D(f) = [1, \infty)$

$R(f) = [1, \infty)$ $\quad R(f) = [2, \infty)$