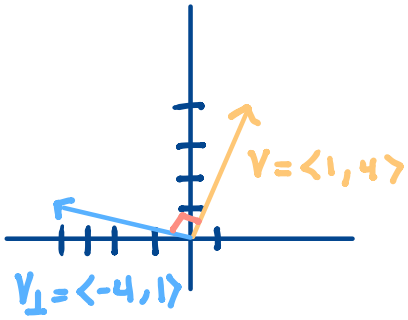
1. Take $V = \langle 1, 4 \rangle$. Determine $V_\perp$.

$V_\perp = \langle -4, 1 \rangle$

Visual



2. If L is the line given by $y = 4x + 1$, write an equation for the line perpendicular to L that passes through $(3, 4)$.

$y = 4x+1$ Slope $m=4$ Vector $V=\langle 1,4\rangle$ $\Rightarrow$ Perpendicular line $V_\perp = \langle -4,1\rangle$ or $m_\perp = -\frac{1}{4}$

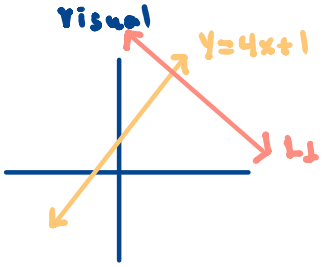
$y = -\frac{1}{4}(x-3)+4$

$= -\frac{1}{4}x + \frac{3}{4} + 4$

$= -\frac{1}{4}x + \frac{19}{4}$

Final answer

Visual



3. Take L to be the line $y = 4x + 1$. Find the point on L that is closest to the point $(3, 4)$.

$L(x) = 4x+1$

$L_\perp(x) = -\frac{1}{4}x + \frac{19}{4}$ line perpendicular to L that passes through $(3,4)$

Determine x so that

$4x+1 = -\frac{1}{4}x + \frac{19}{4}$

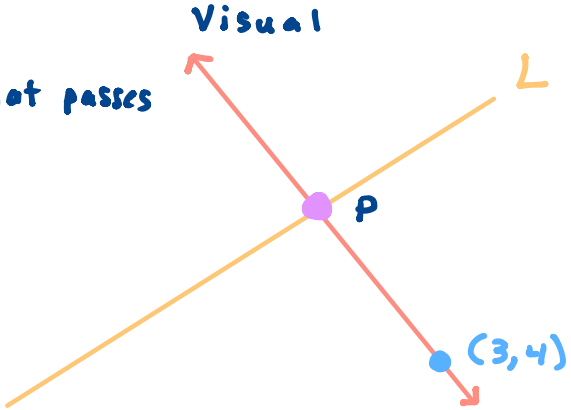
$4x + \frac{1}{4}x = \frac{19}{4} - 1$

$\frac{16}{4}x + \frac{1}{4}x = \frac{19}{4} - \frac{4}{4}$

$\frac{17}{4}x = \frac{15}{4}$

$x = \frac{15}{17}$

Visual

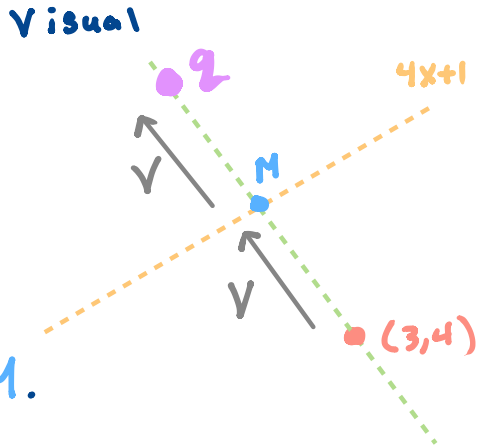


Point P is

$P = \left(\frac{15}{17}, L\left(\frac{15}{17}\right)\right) = \left(\frac{15}{17}, 4\left(\frac{15}{17}\right)+1\right) = \left(\frac{15}{17}, \frac{77}{17}\right)$ final answer

4. Reflect the point $(3, 4)$ across the line $y = 4x + 1$.

Determine M, the point on $L = 4x+1$ closest to $(3,4)$.



The point is

$$M = \left(\frac{15}{17}, \frac{77}{17}\right).$$

Determine vector V that moves $(3,4)$ to M.

$$V = \left(\frac{15}{17}, \frac{77}{17}\right) - (3,4)$$
$$= \left\langle -\frac{36}{17}, \frac{9}{17} \right\rangle.$$

The desired point is

$$q = 2V + (3,4)$$
$$= \left(-\frac{72}{17} + 3, \frac{18}{17} + 4\right)$$
$$= \left(-\frac{21}{17}, \frac{86}{17}\right)$$

final answer