1. Suppose that $(2,4)$ and $(5,6)$ lie on the line L. Find all points on L that are a distance of 2 from $(5,6)$.

$$V = (5,6) - (2,4) = \langle 5-2, 6-4 \rangle = \langle 3, 2 \rangle$$

$$\|V\| = \sqrt{3^2 + 2^2} = \sqrt{9+4} = \sqrt{13}$$

$$\hat{V} = \frac{1}{\|V\|} V = \left\langle \frac{3}{\sqrt{13}}, \frac{2}{\sqrt{13}} \right\rangle \Rightarrow$$

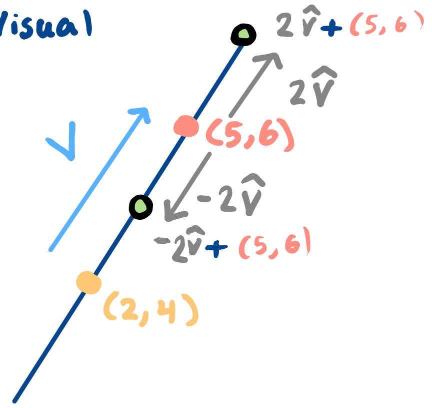
$$2\hat{V} + (5,6) = \left(\frac{6}{\sqrt{13}} + 5, \frac{4}{\sqrt{13}} + 6\right)$$

and

$$-2\hat{V} + (5,6) = \left(-\frac{6}{\sqrt{13}} + 5, -\frac{4}{\sqrt{13}} + 6\right)$$

final answer

Visual



2. A particle that moves at a constant velocity is at $(2,4)$ at 2, moves at a speed of 1, and intersects the point $(5,6)$. Find an equation for the position of the particle at time t.

$$V = (5,6) - (2,4) = \langle 5-2, 6-4 \rangle = \langle 3, 2 \rangle$$

$$\hat{V} = \left\langle \frac{3}{\sqrt{13}}, \frac{2}{\sqrt{13}} \right\rangle$$

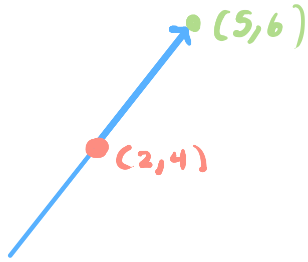
$\hat{V}$ moves points along line that passes $(2,4)$ and $(5,6)$

speed 1

$$\Rightarrow \ell(t) = (t-2)\hat{V} + (2,4)$$

at $(2,4)$ at $t=2$

Visual



3. A particle moves at a constant velocity on $[2,5]$ and $(5,7]$. It is at $(0,1)$ at time 2, at $(1,5)$ at time 5 and at $(-2,5)$ at time 7. Find an equation for the position, $\ell(t)$, of the particle at t.

Path on $[2,5]$

$V_1 = (1,5) - (0,1) = \langle 1, 4 \rangle$

$(0,1)$ at $t=2$

$(1,5)$ at $t=5$

$$\Rightarrow \ell_1(t) = \frac{t-2}{5-2} V_1 + (0,1) = \frac{t-2}{3}\langle 1,4 \rangle + (0,1) = \left\langle \frac{t-2}{3}, 4\left(\frac{t-2}{3}\right) \right\rangle + (0,1) = \left(\frac{t-2}{3}, \frac{4t-8}{3} + 1\right)$$

Path on $(5,7)$

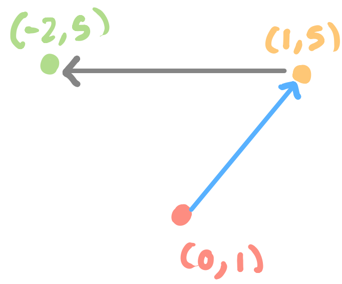
$V_2 = (-2,5) - (1,5) = \langle -3, 0 \rangle$

$(1,5)$ at $t=5$

$(-2,5)$ at $t=7$

$$\Rightarrow \ell_2(t) = \frac{t-5}{7-5} V_2 + (1,5) = \frac{t-5}{2}\langle -3, 0 \rangle + (1,5) = \left\langle -3\left(\frac{t-5}{2}\right), 0 \right\rangle + (1,5) = \left(\frac{-3t+15}{2} + 1, 5\right)$$

Visual

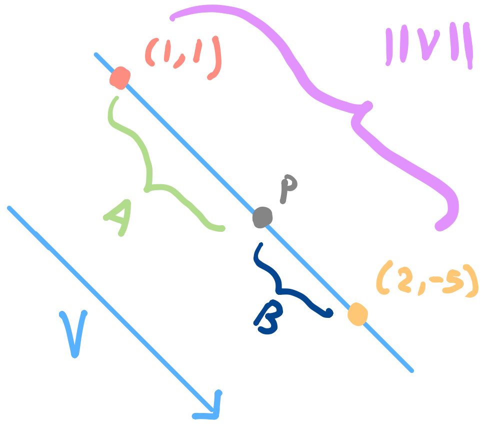


Final answer

$$\ell(t) = \begin{cases} \left(\frac{t-2}{3}, \frac{4t-8}{3} + 1\right) & \text{if } 2 \le t \le 5 \\ \left(\frac{-3t+15}{2} + 1, 5\right) & \text{if } 5 < t \le 7 \end{cases}$$

4. Take L to be a line that passes through $(1, 1)$ and $(2, -5)$. Determine the point p in L so that the distance from $(1, 1)$ to p is three times the distance from p to $(2, -5)$.

Visual



$$V = (2,-5) - (1,1)$$
$$= \langle 1, -6 \rangle$$

$$P = A\langle 1, -6 \rangle + (1,1)$$

$$A = \underbrace{\| p - (1,1) \|}_{}$$ distance from p to (1,1)

$$B = \| (2,-5) - p \|$$ distance from p to (2,-5)

Where A is so that

$$\begin{cases} (\mathrm{I})\ A = 3B & \text{distance from p to (1,1) is three times distance from p to (2,-5)} \\ (\mathrm{II})\ A + B = \|V\| & \text{distance sum up to length of V} \end{cases}$$

$$(\mathrm{I}) \Rightarrow \tfrac{1}{3}A = B$$

$$(\mathrm{II}) \Rightarrow A + B = \|V\|$$
$$A + \tfrac{1}{3}A = \|V\|$$
$$\tfrac{4}{3}A = \|V\|$$
$$A = \tfrac{3}{4}\|V\|$$

Thus

$$P = \tfrac{3}{4}\langle 1, -6 \rangle + (1,1)$$

final answer