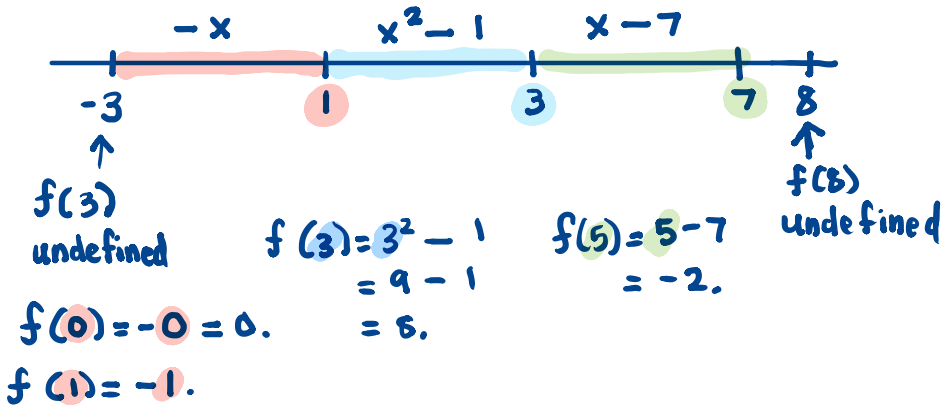
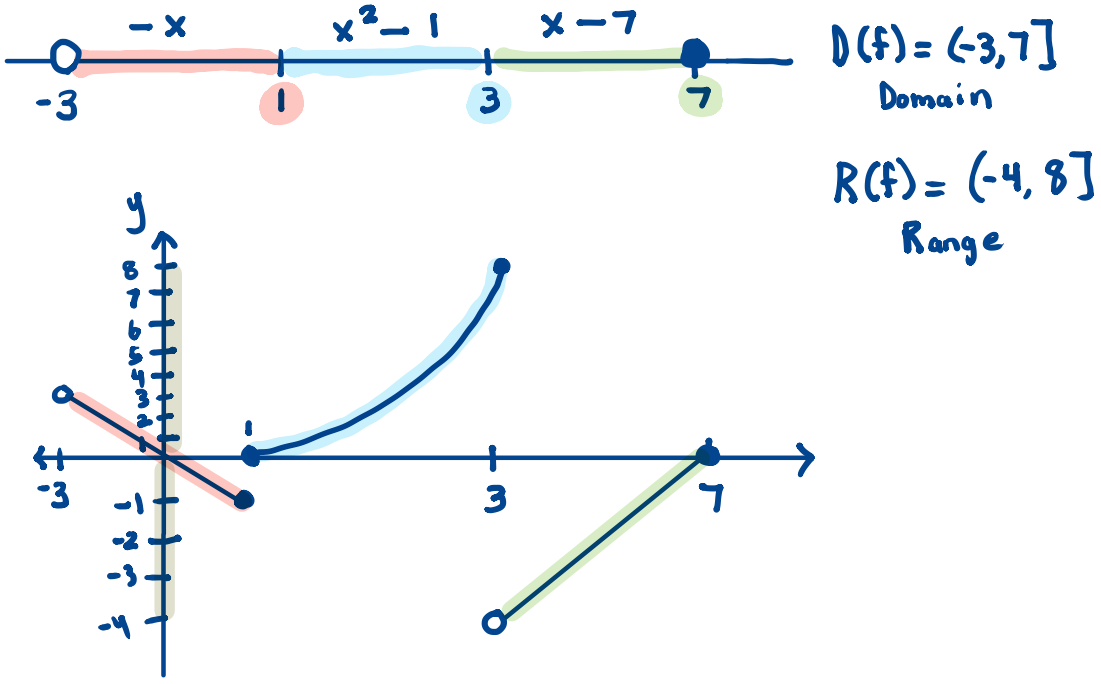
1. Take

$$f(x) = \begin{cases} -x & \text{if } -3 < x \leq 1 \\ x^2 - 1 & \text{if } 1 < x \leq 3 \\ x - 7 & \text{if } 3 < x \leq 7. \end{cases}$$

Evaluate f at $x = -3$, $x = 0$, $x = 1$, $x = 3$, $x = 5$, $x = 8$.



2. Take f as before. Determine the domain of f. Use a sketch of the function f to determine the range of f.

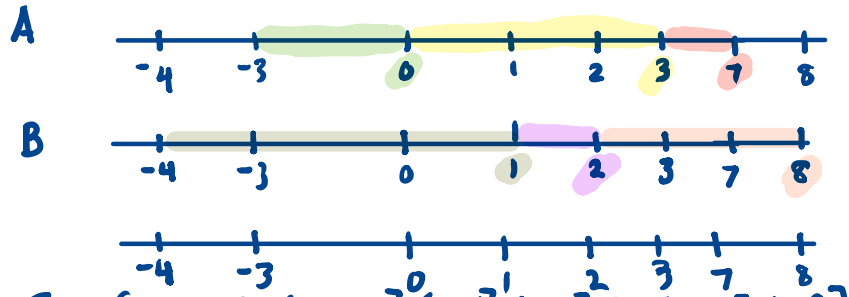


3. Take

$$A = \{(-3, 0], (0, 3], (3, 7]\} \quad \text{and} \quad B = \{(-4, 1], (1, 2], (2, 8]\}.$$

Find a common refinement for A and B.

Many examples, here is one.

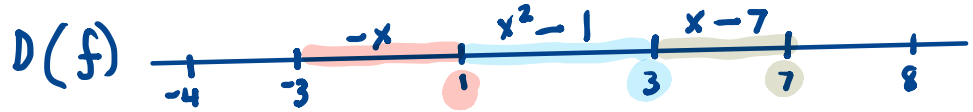


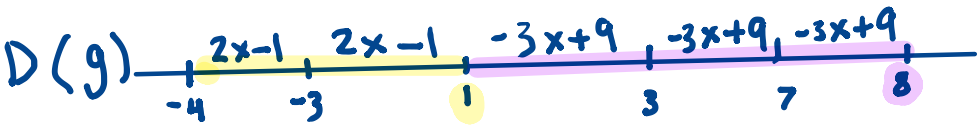
common refinement $C = \{(-4, -3], (-3, 0], (0, 1], (1, 2], (2, 3], (3, 7], (7, 8]\}$

4. Take

$$f(x) = \begin{cases} -x & \text{if } -3 < x \le 1 \\ x^2 - 1 & \text{if } 1 < x \le 3 \\ x - 7 & \text{if } 3 < x \le 7 \end{cases} \quad \text{and} \quad g(x) = \begin{cases} 2x - 1 & \text{if } -4 < x \le 1 \\ -3x + 9 & \text{if } 1 < x \le 8. \end{cases}$$

Determine $\mathcal{D}(f + g)$, $\mathcal{D}(fg)$, and $\mathcal{D}\left(\frac{f}{g}\right)$.





$D(f+g)$ and $D(fg)$ is $(-3, 7]$

$g(x) = 0$ on $(-4, 1]$: $2x - 1 = 0$, $x = \frac{1}{2}$; on $(1, 8]$: $-3x + 9 = 0$, $x = 3$

$D\left(\frac{f}{g}\right)$ is $D(f) \cap D(g) \setminus \{x \in D(g) : g(x) \neq 0\}$

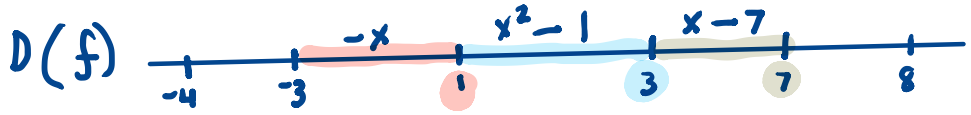
$(-3, 7) \setminus \{\frac{1}{2}, 3\}$

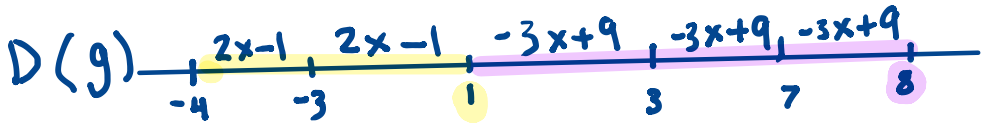
$(-3, \frac{1}{2}) \cup (\frac{1}{2}, 3) \cup (3, 7)$

5. Take

$$f(x)=\begin{cases}-x & \text{if } -3<x\le 1\\ x^2-1 & \text{if } 1<x\le 3\\ x-7 & \text{if } 3<x\le 7\end{cases} \quad \text{and} \quad g(x)=\begin{cases}2x-1 & \text{if } -4<x\le 1\\ -3x+9 & \text{if } 1<x\le 8.\end{cases}$$

Compute $f+g$, fg, and $\frac{f}{g}$.





$f(x)=-x$ $f(x)=x^2-1$ $f(x)=x-7$

$g(x)=2x-1$ $g(x)=-3x+9$ $g(x)=-3x+9$

$$(f+g)(x)=\begin{cases}-x+2x-1 & \text{if } -3<x\le 1\\ x^2-1+-3x+9 & \text{if } 1<x\le 3\\ x-7+-3x+9 & \text{if } 3<x\le 7\end{cases} \text{ or } \begin{cases}x-1 & \text{if } -3<x\le 1\\ x^2-3x+8 & \text{if } 1<x\le 3\\ -2x+2 & \text{if } 3<x\le 7\end{cases}$$

$$(fg)(x)=\begin{cases}-x\cdot(2x-1) & \text{if } -3<x\le 1\\ (x^2-1)\cdot(-3x+9) & \text{if } 1<x\le 3\\ (x-7)\cdot(-3x+9) & \text{if } 3<x\le 7\end{cases} \text{ or } \begin{cases}-2x^2+x & \text{if } -3<x\le 1\\ -3x^3+9x^2+3x+9 & \text{if } 1<x\le 3\\ -3x^2+30x-63 & \text{if } 3<x\le 7\end{cases}$$

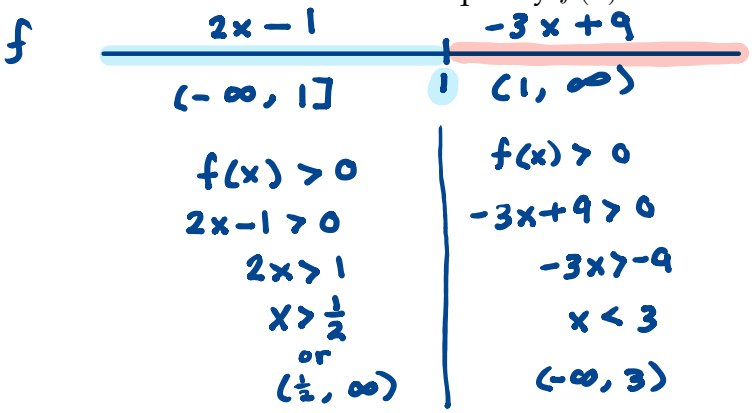
$g(x)=0$ on $\{\frac{1}{2},3\}$, so

$$\left(\frac{f}{g}\right)(x)=\begin{cases}\frac{-x}{2x-1} & \text{if } -3<x<\frac{1}{2}\\ \frac{-x}{2x-1} & \text{if } \frac{1}{2}<x\le 1\\ \frac{x^2-1}{-3x+9} & \text{if } 1<x<3\\ \frac{x-7}{-3x+9} & \text{if } 3<x<7\end{cases}$$

6. Take

$$f(x) = \begin{cases} 2x - 1 & \text{if } x \leq 1 \\ -3x + 9 & \text{if } x > 1. \end{cases}$$

Sketch on a number line the solution set to the inequality $f(x) > 0$.



$f(x) > 0$
$2x - 1 > 0$
$2x > 1$
$x > \frac{1}{2}$
or
$(\frac{1}{2}, \infty)$

$f(x) > 0$
$-3x + 9 > 0$
$-3x > -9$
$x < 3$
$(-\infty, 3)$

$\Rightarrow ((-\infty, 1] \cap (\frac{1}{2}, \infty)) \cup ((-\infty, 3) \cap (1, \infty))$

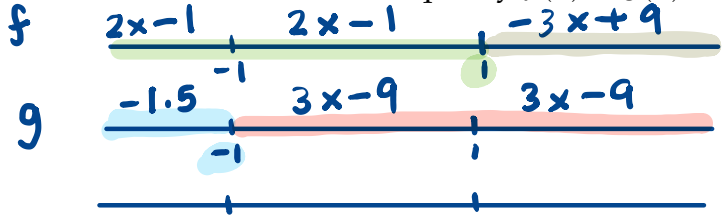
$(\frac{1}{2}, 1] \cup (1, 3)$

$(\frac{1}{2}, 3)$ final answer

7. Take

$$f(x) = \begin{cases} 2x - 1 & \text{if } x \leq 1 \\ -3x + 9 & \text{if } x > 1 \end{cases} \quad \text{and} \quad g(x) = \begin{cases} -1.5 & \text{if } x \leq -1 \\ 3x - 9 & \text{if } x > -1. \end{cases}.$$

Sketch on a number line the solution set to the inequality $f(x) > g(x)$.



$(-\infty, -1]$ | $(-1, 1)$ | $[1, \infty)$

$f(x) > g(x)$
$2x - 1 > -1.5$
$2x > -0.5$
$x > -0.25$
or
$x > -\frac{1}{4}$

$f(x) > g(x)$
$2x - 1 > 3x - 9$
$8 > x$
or
$x < 8$

$f(x) > g(x)$
$-3x + 9 > 3x - 9$
$18 > 6x$
$3 > x$
or
$x < 3$

$\Rightarrow ((-\infty, -1] \cap (-\frac{1}{4}, \infty)) \cup ((-1, 1) \cap (-\infty, 8)) \cup ([1, \infty) \cap (-\infty, 3))$

$\{\ \} \cup (-1, 1) \cup [1, 3)$

$(-1, 3)$ final answer

8. Sketch on a number line the solution set to the inequality

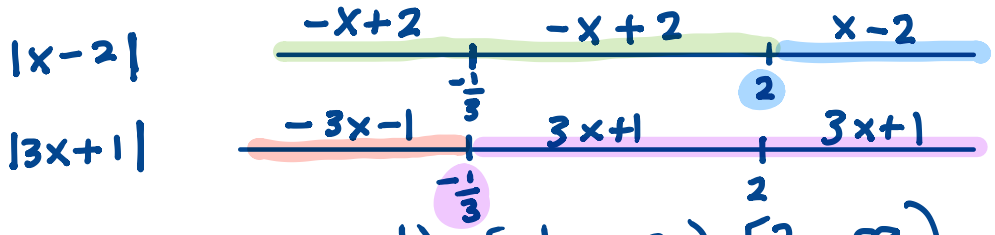
$$|x-2| > |3x+1|.$$

Note $|A| = \begin{cases} A & \text{if } A \geq 0 \\ -A & \text{if } A < 0 \end{cases}$.

So

$|x-2| = \begin{cases} x-2 & \text{if } x-2 \geq 0 \\ -(x-2) & \text{if } x-2 < 0 \end{cases}$ and $|3x+1| = \begin{cases} 3x+1 & \text{if } 3x+1 \geq 0 \\ -(3x+1) & \text{if } 3x+1 < 0 \end{cases}$

$= \begin{cases} x-2 & \text{if } x \geq 2 \\ -x+2 & \text{if } x < 2 \end{cases}$ $= \begin{cases} 3x+1 & \text{if } x \geq -\frac{1}{3} \\ -3x-1 & \text{if } x < -\frac{1}{3} \end{cases}$



$(-\infty, -\frac{1}{3})$	$[-\frac{1}{3}, 2)$	$[2, \infty)$
$\lvert x-2\rvert > \lvert 3x+1\rvert$	$\lvert x-2\rvert > \lvert 3x+1\rvert$	$\lvert x-2\rvert > \lvert 3x+1\rvert$
$-x+2 > -3x-1$	$-x+2 > 3x+1$	$x-2 > 3x+1$
$2x > -3$	$1 > 4x$	$-3 > 2x$
$x > -3/2$	$\frac{1}{4} > x$ or $x < \frac{1}{4}$	$-\frac{3}{2} > x$ or $x < -3/2$

$\Rightarrow \left((-\infty, -\frac{1}{3}) \cap (-\frac{3}{2}, \infty)\right) \cup \left([-\frac{1}{3}, 2) \cap (-\infty, \frac{1}{4})\right) \cup \left([2, \infty) \cap (-\infty, -\frac{3}{2})\right)$

$(-\frac{3}{2}, -\frac{1}{3}) \cup [-\frac{1}{3}, -\frac{1}{4}) \cup \{\}$

$\left(-\frac{3}{2}, -\frac{1}{4}\right)$

final answer