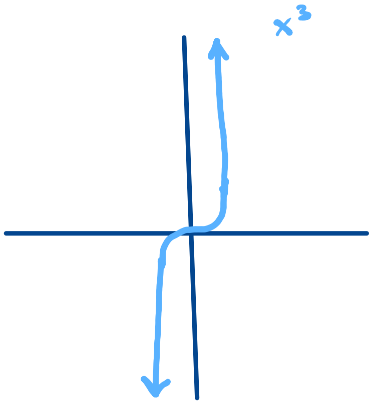
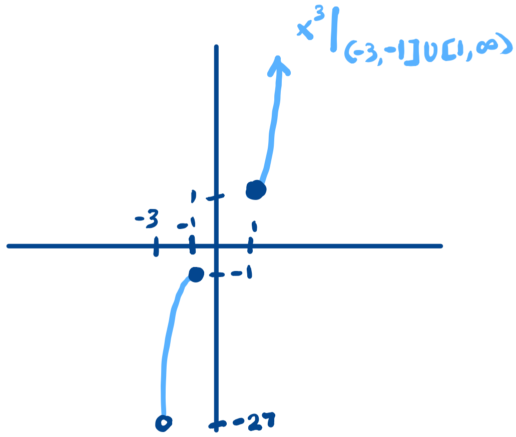
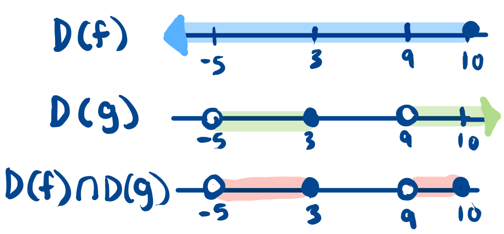
1. Take $f(x) = x^3$. Draw the function on the restriction $(-3, -1] \cup [1, \infty)$.





2. Take f and g to be functions with $\mathcal{D}(f) = (-\infty, 10]$, $\mathcal{D}(g) = (-5, 3] \cup (9, \infty)$, and the zero set of g is $\{3, 11\}$. Determine the domain of $f + g$, fg, and $\frac{f}{g}$.



Domain of $f+g$ and fg is $(-5, 3] \cup (9, 10]$

Domain of $\frac{f}{g}$ is

zero set of g

$D(f) \cap D(g) \setminus \{3, 11\}$
$(-5, 3) \cup (9, 10]$

3. Determine how $f - g$ is defined and the domain of $f - g$.

$(f-g)(x) = f(x) - g(x)$, domain of $f - g$ is

definition of $f-g$ $D(f) \cap D(g)$

4. Take f and g to be given by

$$f(x) = 2x \quad \text{and} \quad g(x) = \sqrt{x - 4}.$$

Determine a formula for $f \circ g$ and $g \circ f$. Also determine the domain of both functions.

- $(f \circ g)(x) = f(g(x))$
  $= 2(g(x))$
  $= 2\sqrt{x-4}$ $\Rightarrow$ $\boxed{(f \circ g)(x) = 2\sqrt{x-4}}$

domain of $f \circ g$
need $x - 4 \geq 0$

$\boxed{[4, \infty) \text{ domain of } f \circ g}$

- $(g \circ f)(x) = g(f(x))$
  $= \sqrt{f(x) - 4}$ $\Rightarrow$ $\boxed{(g \circ f)(x) = \sqrt{2x-4}}$
  $= \sqrt{2x - 4}$

domain of $g \circ f$
need $2x - 4 \geq 0$

$\boxed{[2, \infty) \text{ domain of } g \circ f}$

5. Take

$$a(x) = x, \quad b(x) = x^2, \quad c(x) = x+3, \quad d(x) = 5x, \quad \text{and} \quad e(x) = \sqrt{x}.$$

Decompose f into sums, products, quotients and or composites of more elementary functions, where

$$f(x) = x^2\sqrt{x+5x^2} + \frac{x+3}{x}.$$

① $x^2 = b$
$5x = d$ $\Rightarrow 5x^2 = (d \circ b)(x)$

② $x = a$
$5x^2 = d \circ b$ $\Rightarrow x + 5x^2 = a + d \circ b$

③ $\sqrt{x} = e$
$x + 5x^2 = a + d \circ b$ $\Rightarrow \sqrt{x+5x^2} = e \circ (a + d \circ b)$

$$\boxed{b \cdot e \circ (a + d \circ b) + \frac{c}{a}}$$

6. Take $f(x) = \frac{4x+1}{x-3}$. Determine the domain and range of f.

Domain:

Can't divide by zero

$x - 3 = 0$

$x = 3$

$$\boxed{D(f) = (-\infty, 3) \cup (3, \infty)}$$

Range:

Determine all real numbers b so that

$f(x) = b$

has a solution.

$f(x) = b$

$\frac{4x+1}{x-3} = b$

$4x + 1 = b(x-3)$

$4x + 1 = bx - 3b$

$4x - bx = -3b - 1$

$(4-b)x = -3b - 1$

$x = \frac{-3b-1}{4-b}$, undefined if $4 - b = 0$, so $b = 4$.

$$\boxed{R(f) = (-\infty, 4) \cup (4, \infty)}$$