

1. Explicitly write down all the elements of the set  $S$  given by

$$S = \{n \in \mathbb{N} : 2n \leq 8\}.$$

All natural numbers  $n$  with the property that  $2n$  is less than or equal to 8.

$$\begin{aligned} 1 &\Rightarrow 2 \cdot 1 = 2 \\ 2 &\Rightarrow 2 \cdot 2 = 4 \\ 3 &\Rightarrow 2 \cdot 3 = 6 \\ 4 &\Rightarrow 2 \cdot 4 = 8 \end{aligned}$$

$$S = \{1, 2, 3, 4\}$$

2. Take  $A$  and  $B$  to be the sets given by

$$A = \{-1, 1\} \quad \text{and} \quad B = \{-1, 0, 1\}.$$

Determine whether the following statements are true or false.

a)  $A \subseteq B$

True. Every element in  $A$  is also an element in  $B$ .

b)  $B \subseteq A$

False. The number 0 is an element of  $B$  that is not an element in  $A$ .

3. Take  $A$ ,  $B$  and  $C$  to be the sets given by

$$A = \{-1, 1\}, \quad B = \{-1, 0, 1\} \quad \text{and} \quad C = \{0, 1, 5, 6\}.$$

Determine whether the following statements are true or false.

a)  $A \subseteq B$

True

b)  $B \subseteq A$

False  
0 is in  $B$  but not in  $A$

c)  $C \subseteq A$

False  
5 is in  $C$  but not in  $A$

d)  $B \subseteq C$

False  
-1 is in  $B$  but not in  $C$

e)  $C \subseteq B$

False  
5 is in  $C$  but not in  $B$

4. Take  $A$  and  $B$  to be the sets given by

$$A = \{-2, -1, 0, 1, 4\} \quad \text{and} \quad B = \{-1, 0, 1, 3\}.$$

Write out explicitly all elements.

a)  $A \cup B$

union

$$A \cup B = \{-2, -1, 0, 1, 3, 4\}$$

b)  $B \cap A$

intersection

$$B \cap A = \{-1, 0, 1\}$$

5. Determine the following intersection:

$$\{1\} \cap \{-1, 0\}.$$

empty

$$\emptyset \text{ or } \{\}$$

6. Take  $X, Y$  and  $Z$  to be the sets given by

$$X = \{x \in \mathbb{N} : x \text{ is even}\}, \quad Y = \{y \in \mathbb{N} : y < 21\} \quad \text{and} \quad Z = \{z \in \mathbb{N} : z \text{ is a multiple of } 3\}.$$

Determine

$$Z \cap (X \cap Y).$$

① Write out all elements.

$$\begin{aligned} X &= \{2, 4, 6, 8, 10, \dots\} \\ Y &= \{1, 2, 3, 4, \dots, 19, 20\} \\ Z &= \{3, 6, 9, 12, \dots\} \end{aligned}$$

② Compute  $(X \cap Y)$

$$X \cap Y = \{2, 4, 6, \dots, 18, 20\}$$

③ compute  $Z \cap (X \cap Y)$

$$\begin{aligned} Z \cap (X \cap Y) &= \{3, 6, 9, 12, 15, \dots\} \cap \{2, 4, 6, \dots, 18, 20\} \\ &= \{6, 12, 18\} \end{aligned}$$

7. Take  $A$  and  $B$  to be the sets given by

$$A = \{-2, -1, 0, 1, 2\} \quad \text{and} \quad B = \{-3, -1, 0, 3\}.$$

Determine the following.

a)  $A \setminus B$

all elements in  $A$  not in  $B$

$$A \setminus B = \{-2, 1, 2\}$$

b)  $B \setminus A$

all elements in  $B$  not in  $A$

$$B \setminus A = \{-3, 3\}$$